Pulse Shaping

Two conflicting objectives:

1. Improve Signal-to-Noise Ratio S/N

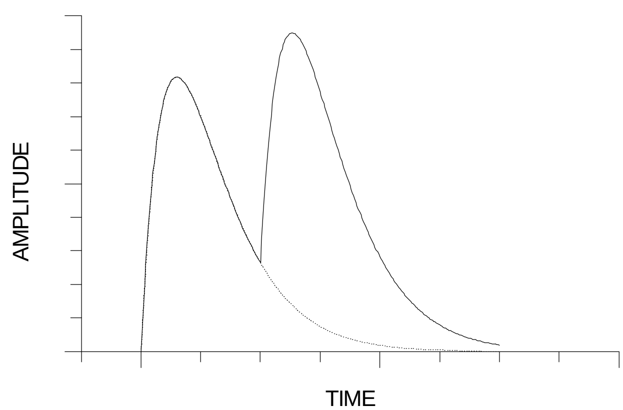
   Restrict bandwidth to match measurement time

   $\Rightarrow$ Increase Pulse Width

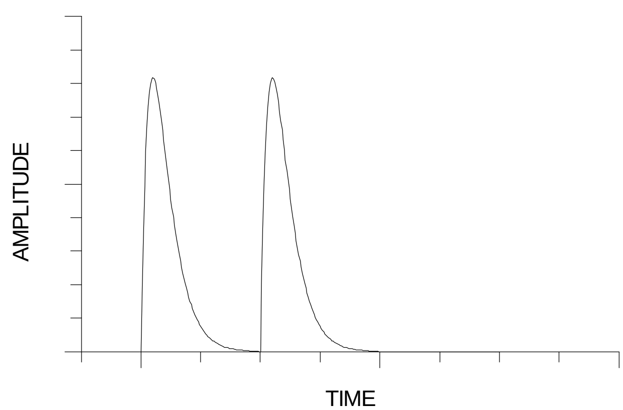
2. Improve Pulse Pair Resolution

   $\Rightarrow$ Decrease Pulse Width

Pulse pile-up distorts amplitude measurement



Reducing pulse shaping time to 1/3 eliminates pile-up.

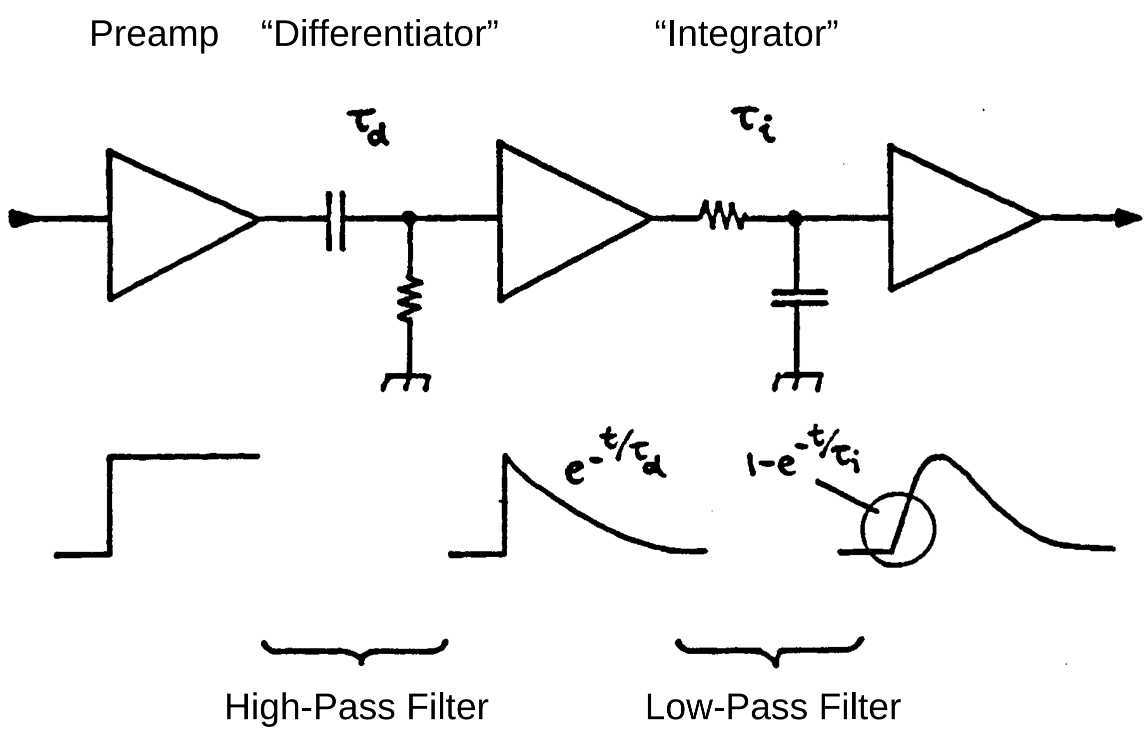


Necessary to find balance between these conflicting requirements. Sometimes minimum noise is crucial, sometimes rate capability is paramount.

Usually, many considerations combined lead to a "non-textbook" compromise.

- *"Optimum shaping" depends on the application!*
- Shapers need not be complicated –
  Every amplifier is a pulse shaper!

Simple Example: CR-RC Shaping



Simple arrangement: Noise performance only 36% worse than optimum filter with same time constants.

⇒ Useful for estimates, since simple to evaluate

CR Differentiator

Input voltage = voltage across C + voltage across R

$$\frac{dV_{in}(t)}{dt} = \frac{1}{C}\frac{dQ}{dt} + \frac{dV_R(t)}{dt}$$

Using $V_R(t) = R \cdot i(t)$ and $V_{out}(t) = V_R(t)$

$$\frac{dV_{in}(t)}{dt} = \frac{1}{C} i(t) + \frac{dV_{out}(t)}{dt} = \frac{1}{C}\frac{V_{out}(t)}{R} + \frac{dV_{out}(t)}{dt}$$

Setting $\tau = RC$

$$V_{out}(t) + \tau \frac{dV_{out}(t)}{dt} = \tau \frac{dV_{in}(t)}{dt}$$

If

$$\tau \frac{dV_{out}(t)}{dt} << V_{out}(t)$$

then

$$V_{out}(t) = \tau \frac{dV_{in}(t)}{dt}$$

i.e. the output is the time derivative of the input.
In practice, this condition is seldom met, but the circuit is still called a "differentiator".

For a step input $V_{in}(t) = 0$ for $t < 0$
$V_{in}(t) = V_i$ for $t \geq 0$

$$V_{out} = V_i \, e^{-t/\tau}$$

i.e. the differentiator *shortens* the pulse (decreases the fall time)

In the frequency domain

$$V_{out} = \frac{R}{R + X_C}\; V_{in} = \frac{R}{R - \dfrac{i}{\omega C}}\; V_{in}$$

$$V_{out} = \frac{i\omega RC}{1 + i\omega RC}\; V_{in} = \frac{1}{1 + 1/\, i\omega\tau}\; V_{in}$$

At low frequencies $\omega << 1/\tau$ ($f << 1/(2\pi\tau)$)

$$V_{out} \approx i\omega\tau \cdot V_{in}$$

At high frequencies, $\omega >> 1/\tau$

$$V_{out} \approx V_{in}$$

⇒ the CR differentiator is a “high-pass” filter, i.e. it transmits frequencies above the cutoff frequency $1/2\pi RC$.

A similar treatment applies to the RC “integrator”, which in the time domain increases the rise time and in the frequency domain acts as a low-pass filter..

Pulse Shaping and Signal-to-Noise Ratio

Pulse shaping affects both the

- total noise

and

- peak signal amplitude

at the output of the shaper.

Equivalent Noise Charge

Inject known signal charge into preamp input
(either via test input or known energy in detector).

Determine signal-to-noise ratio at shaper output.

Equivalent Noise Charge $\equiv$ Input charge for which $S/N = 1$

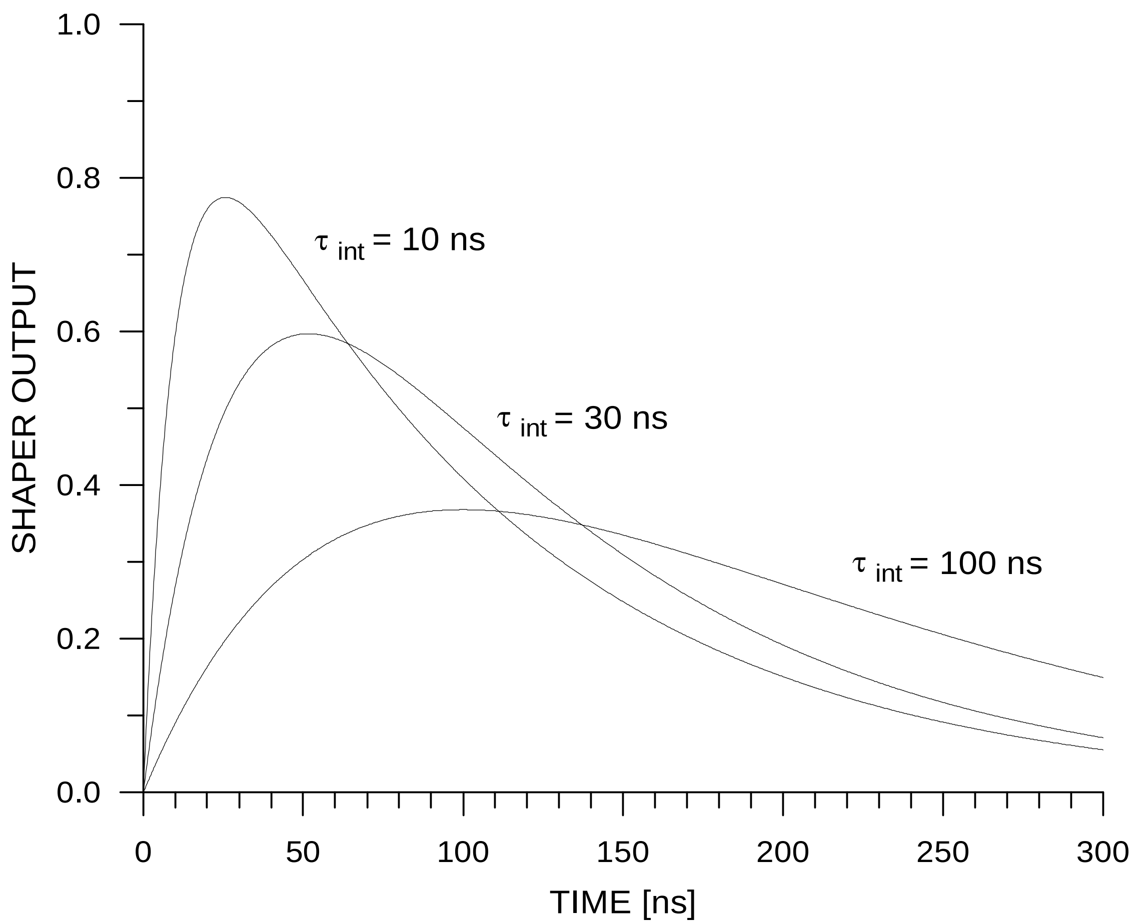
Effect of relative constants

Consider a *CR-RC* shaper with a fixed differentiator time constant of 100 ns.

Increasing the integrator time constant lowers the upper cut-off frequency, which decreases the total noise at the shaper output.

However, as shown in the following figure, the peak signal also decreases.

CR-RC SHAPER
FIXED DIFFERENTIATOR TIME CONSTANT = 100 ns
INTEGRATOR TIME CONSTANT = 10, 30 and 100 ns



Still keeping the differentiator time constant fixed at 100 ns, the next set of graphs shows the variation of

output noise

output signal amplitude

equivalent input noise charge

as the integrator time constant is increased from 10 to 100 ns:

Output Noise:

$$\frac{v_{no}(100\text{ ns})}{v_{no}(10\text{ ns})} = \frac{1}{4.2}$$

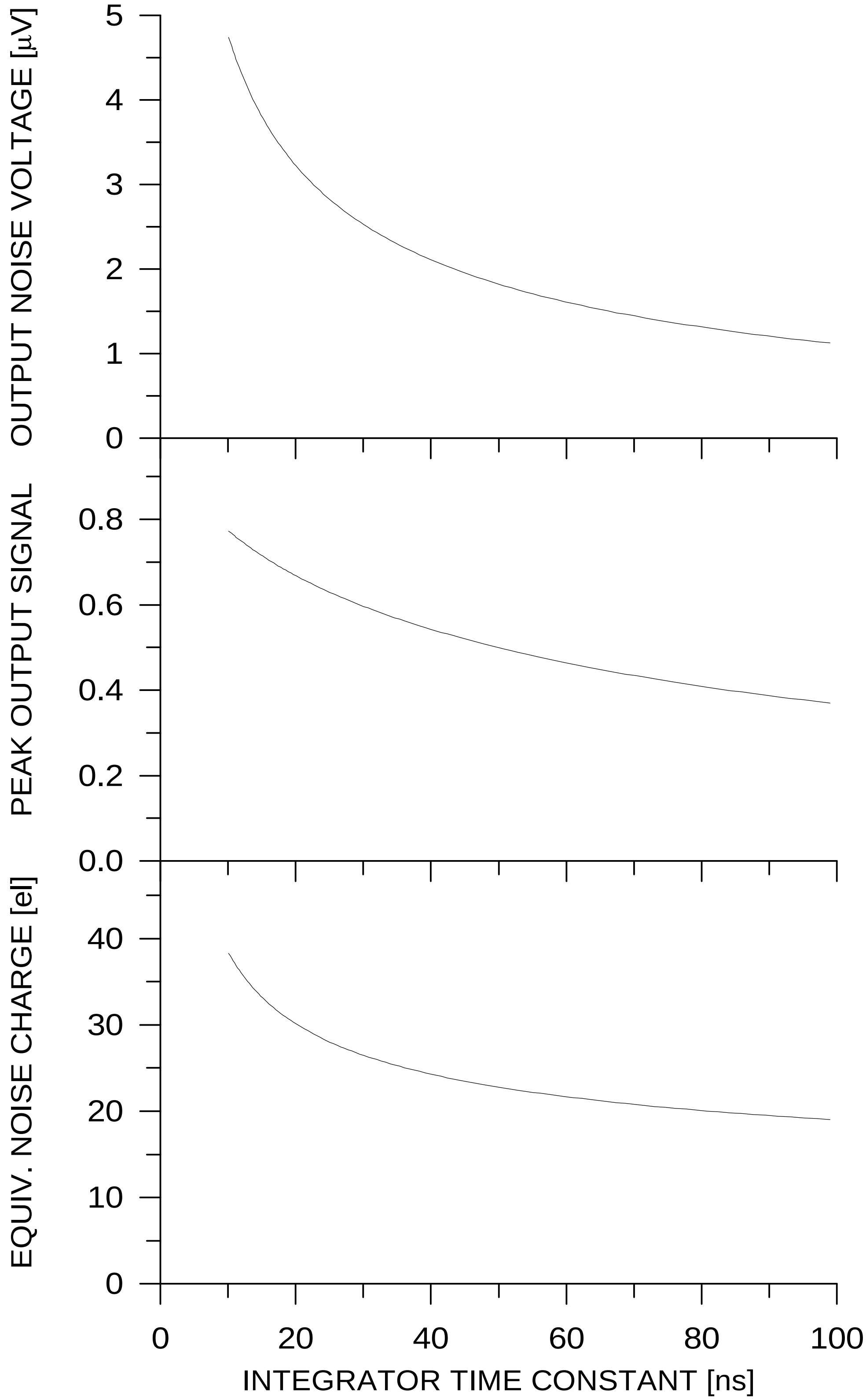
Peak Output Signal:

$$\frac{V_{so}(100\text{ ns})}{V_{so}(10\text{ ns})} = \frac{1}{2.1}$$

The roughly 4-fold decrease in noise is partially compensated by the 2-fold reduction in signal, so that

$$\frac{Q_n(100\text{ ns})}{Q_n(10\text{ ns})} = \frac{1}{2}$$

OUTPUT NOISE, OUTPUT SIGNAL AND EQUIVALENT NOISE CHARGE
CR-RC SHAPER - FIXED DIFFERENTIATOR TIME CONSTANT = 100 ns
(e_n = 1 nV/ √ Hz, i_n = 0, C_{TOT} = 1 pF)



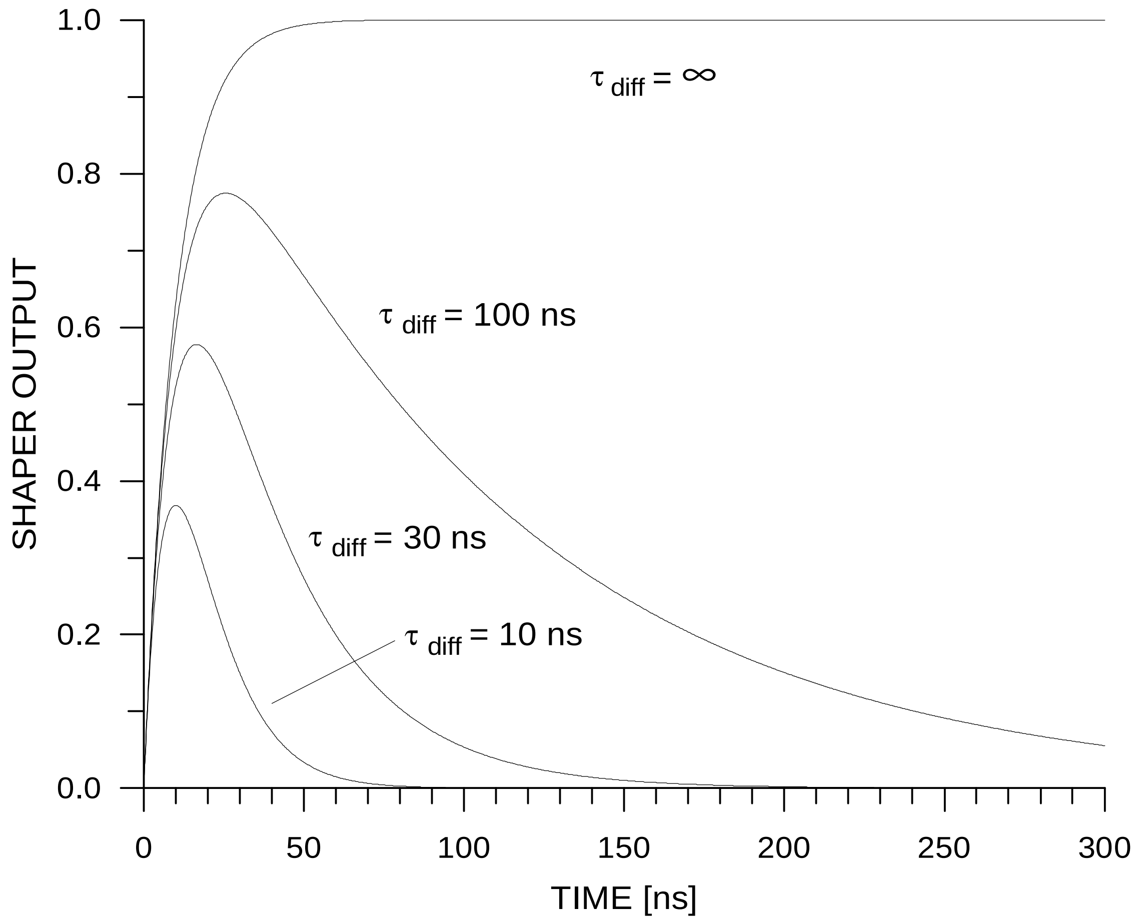
For comparison, consider the same *CR-RC* shaper with the integrator time constant fixed at 10 ns and the differentiator time constant variable.

As the differentiator time constant is changed, the peak signal amplitude at the shaper output varies as shown in the following graph.

Note that the need to limit the pulse width incurs a significant reduction in the output signal.

Even at a differentiator time constant $\tau_{diff} = 100$ ns = 10 τ_{int} the output signal is only 80% of the value for $\tau_{diff} = \infty$, i.e. a system with no low-frequency roll-off.

CR-RC SHAPER
FIXED INTEGRATOR TIME CONSTANT = 10 ns
DIFFERENTIATOR TIME CONSTANT = ∞ , 100, 30 and 10 ns



Keeping the integrator time constant fixed at 10 ns,
the next graph shows

output noise

output signal amplitude

equivalent input noise charge

as the differentiator time constant is changed from 10 to 100 ns.

Since changing the low-frequency cut-off does not affect the total noise bandwidth appreciably, the change in output noise is modest

$$\frac{v_{no}(100\ \mathrm{ns})}{v_{no}(10\ \mathrm{ns})} = 1.3\ ,$$

whereas the signal amplitude changes appreciably.

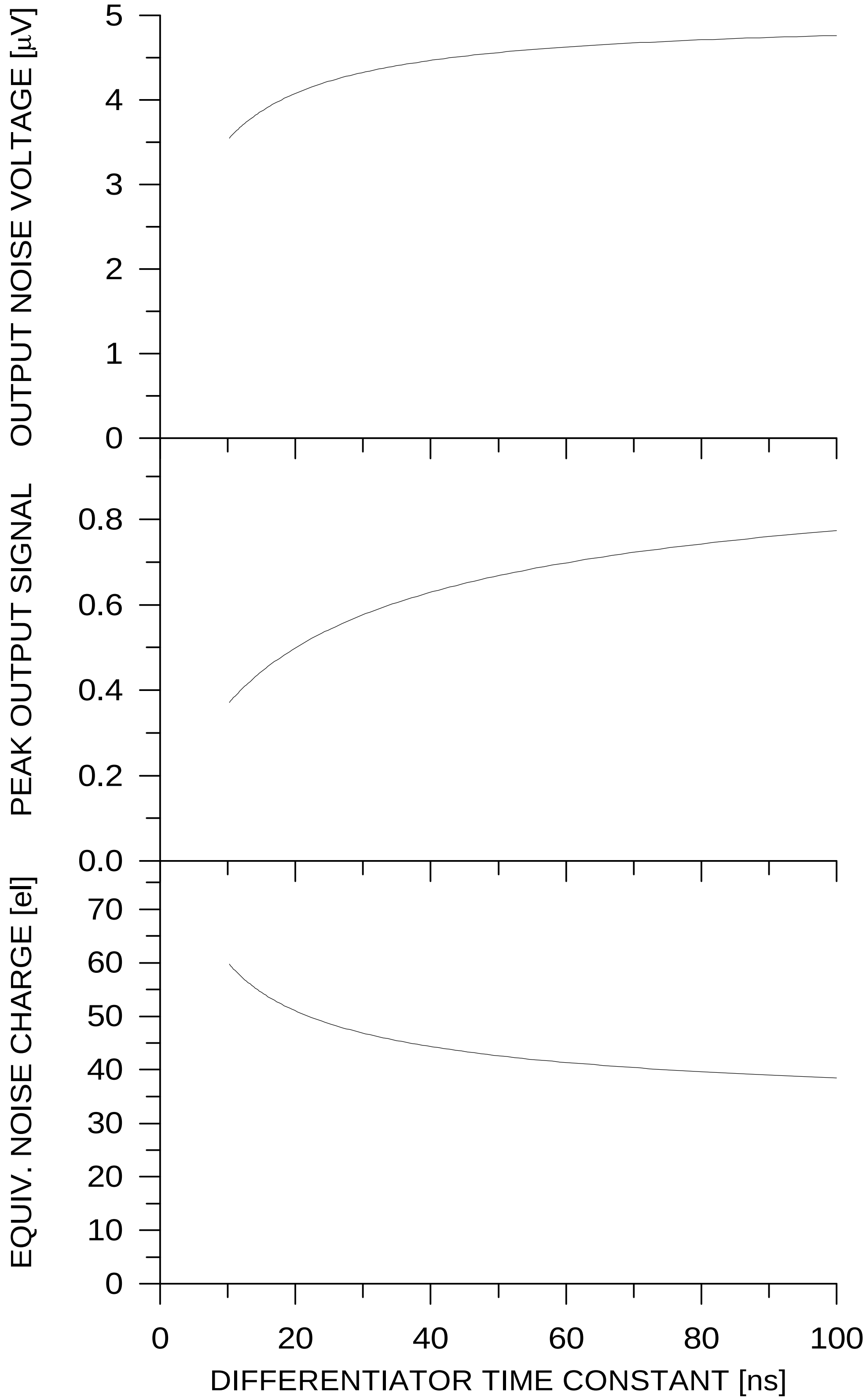
$$\frac{V_{so}(100\ \mathrm{ns})}{V_{so}(10\ \mathrm{ns})} = 2.1$$

Although the noise grows as the differentiator time constant is changed from 10 to 100 ns, it is outweighed by the increase in signal level so that the net signal-to-noise ratio improves.

The equivalent input noise charge

$$\frac{Q_n(100\ \mathrm{ns})}{Q_n(10\ \mathrm{ns})} = \frac{1}{1.6}$$

OUTPUT NOISE, OUTPUT SIGNAL AND EQUIVALENT NOISE CHARGE
CR-RC SHAPER - FIXED INTEGRATOR TIME CONSTANT = 10 ns
(e_n = 1 nV/ √ Hz, i_n = 0, C_{TOT} = 1 pF)



Summary

To evaluate shaper noise performance

- Noise spectrum alone is inadequate

Must also

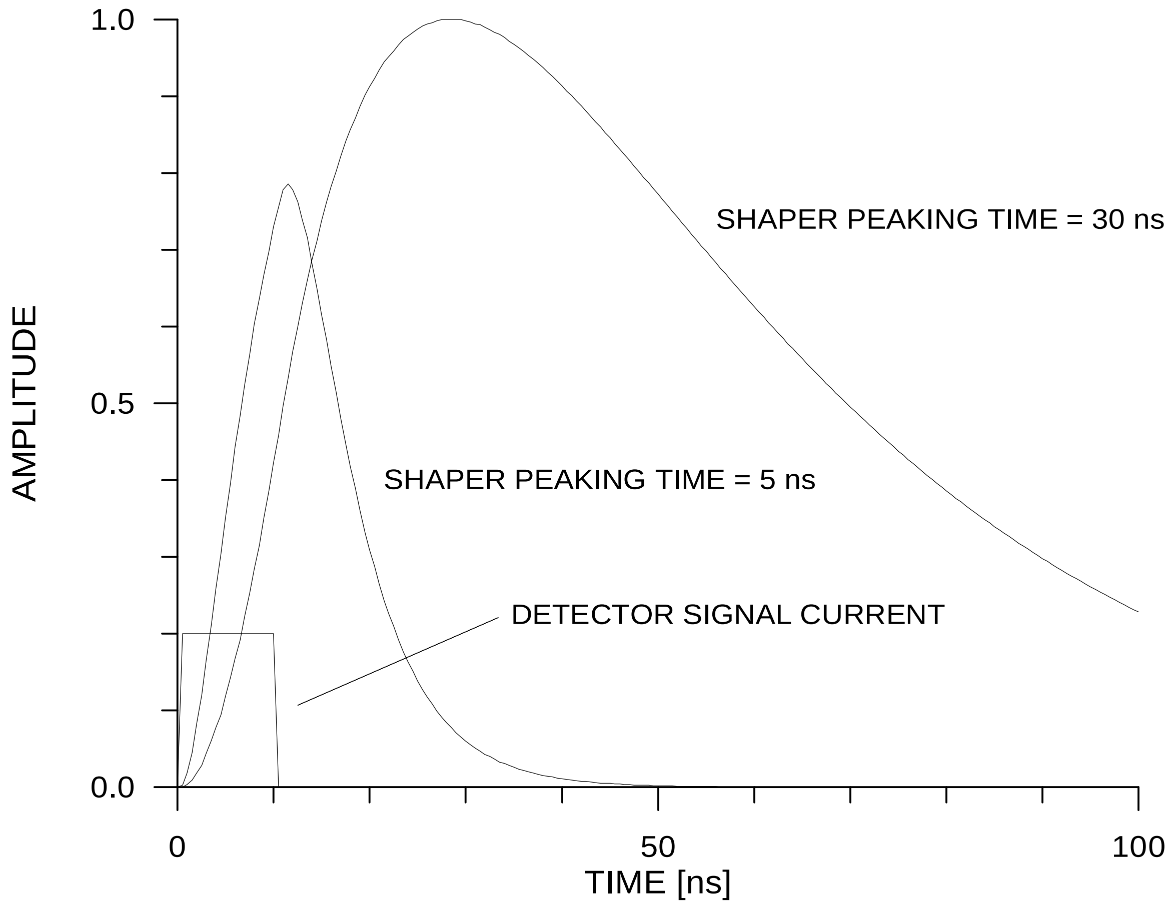
- Assess effect on signal

Signal amplitude is also affected by the relationship of the shaping time to the detector signal duration.

If peaking time of shaper < collection time

$\Rightarrow$ signal loss (“ballistic deficit”)

Loss in Pulse Height (and Signal-to-Noise Ratio) if Peaking Time of Shaper < Detector Collection Time



Note that although the faster shaper has a peaking time of 5 ns, the response to the detector signal peaks after full charge collection.

Evaluation of Equivalent Noise Charge

A. Experiment

Inject an input signal with known charge using a pulse generator set to approximate the detector signal (possible ballistic deficit). Measure the pulse height spectrum.

peak centroid	$\Rightarrow$	signal magnitude
peak width	$\Rightarrow$	noise (FWHM= 2.35 rms)

If pulse-height digitization is not practical:

1. Measure total noise at output of pulse shaper

   a) measure the total noise power with an rms voltmeter of sufficient bandwidth
   or
   b) measure the spectral distribution with a spectrum analyzer and integrate (the spectrum analyzer provides discrete measurement values in N frequency bins Δf_n)

$$V_{no} = \sqrt{\sum_{n=0}^{N} \left(v_{no}^2(n) \cdot \Delta f \right)}$$

   The spectrum analyzer shows if "pathological" features are present in the noise spectrum.

2. Measure the magnitude of the output signal V_{so} for a known input signal, either from detector or from a pulse generator set up to approximate the detector signal.

3. Determine signal-to-noise ratio $S/N = V_{so} / V_{no}$

   and scale to obtain the equivalent noise charge

$$Q_n = \frac{V_{no}}{V_{so}} Q_s$$

B. Numerical Simulation (e.g. SPICE)

This can be done with the full circuit including all extraneous components. Procedure analogous to measurement.

1. Calculate the spectral distribution and integrate

$$V_{no} = \sqrt{\sum_{n=0}^{N} v_{no}^2(n) \cdot \Delta f}$$

2. Determine the magnitude of output signal V_{so} for an input that approximates the detector signal.

3. Calculate the equivalent noise charge

$$Q_n = \frac{V_{no}}{V_{so}} Q_s$$

C. Analytical Simulation

1. Identify individual noise sources and refer to input
2. Determine the spectral distribution at input for each source k

$$v_{ni,k}^2(f)$$

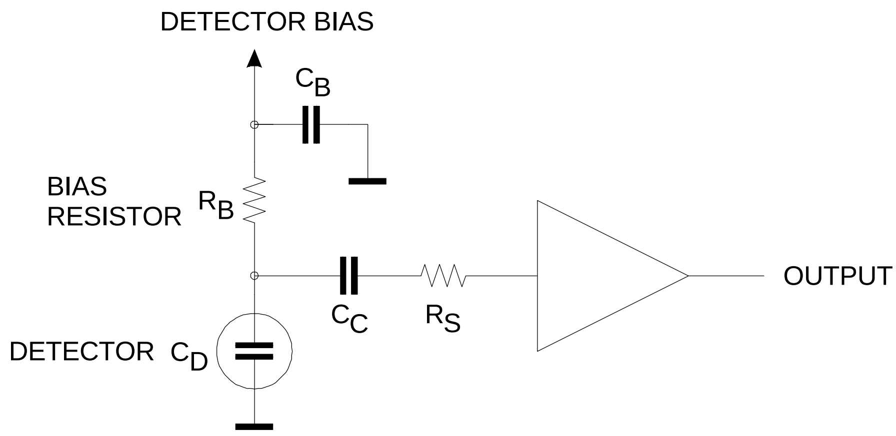
3. Calculate the total noise at shaper output ($G(f)$ = gain)

$$V_{no} = \sqrt{\int_0^\infty G^2(f)\left(\sum_k v_{ni,k}^2(f)\right)df} \equiv \sqrt{\int_0^\infty G^2(\omega)\left(\sum_k v_{ni,k}^2(\omega)\right)d\omega}$$

4. Determine the signal output V_{so} for a known input charge Q_s and realistic detector pulse shape.
5. Equivalent noise charge

$$Q_n = \frac{V_{no}}{V_{so}} Q_s$$

Analytical Analysis of a Detector Front-End



Detector bias voltage is applied through the resistor R_B. The bypass capacitor C_B serves to shunt any external interference coming through the bias supply line to ground. For AC signals this capacitor connects the "far end" of the bias resistor to ground, so that R_B appears to be in parallel with the detector.

The coupling capacitor C_C in the amplifier input path blocks the detector bias voltage from the amplifier input (which is why a capacitor serving this role is also called a "blocking capacitor").

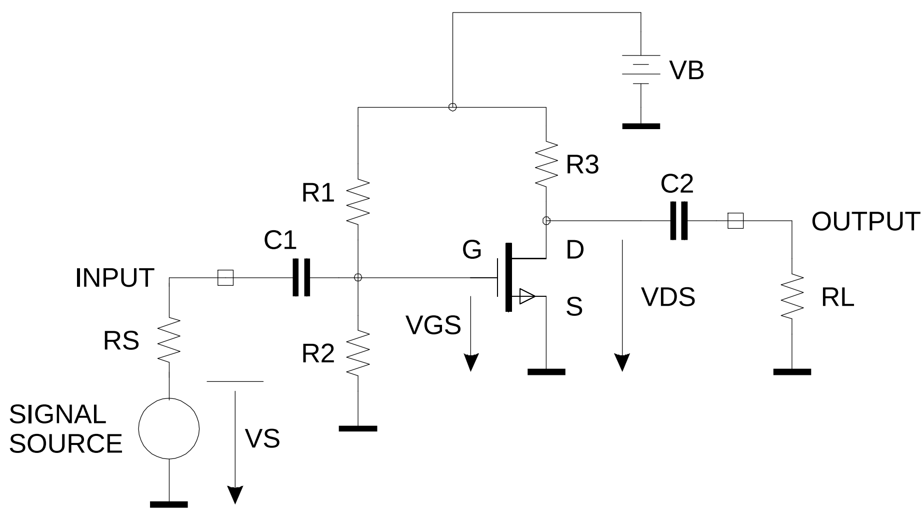
The series resistor R_S represents any resistance present in the connection from the detector to the amplifier input. This includes

- the resistance of the detector electrodes
- the resistance of the connecting wires
- any resistors used to protect the amplifier against large voltage transients ("input protection")
- ... etc.

Equivalent Circuits

Take a simple amplifier as an example.

a) full circuit diagram



First, just consider the DC operating point of the circuitry between $C1$ and $C2$:

1. The n-type MOSFET requires a positive voltage applied from the gate G to the source S.

$$V_{GS} = \frac{R2}{R1 + R2} V_B$$

2. The gate voltage V_{GS} sets the current flowing into the drain electrode D.

3. Assume the drain current is I_D. Then the DC voltage at the drain is

$$V_{DS} = V_B - I_D R3$$

Next, consider the AC signal V_S provided by the signal source.

Assume that the signal at the gate G is dV_G/dt.

1. The current flowing through *R2* is

$$\frac{dI}{dt}(R2) = \frac{dV_G}{dt} \cdot \frac{1}{R2}$$

2. The current flowing through *R1* is

$$\frac{dI}{dt}(R1) = \frac{1}{R1} \cdot \frac{d}{dt}(V_G + \mathrm{V_B})$$

Since the battery voltage $\mathrm{V_B}$ is constant,

$$\frac{d\mathrm{V_B}}{dt} = 0$$

so that

$$\frac{dI}{dt}(R1) = \frac{1}{R1} \cdot \frac{dV_G}{dt}$$

3. The total time-dependent input current is

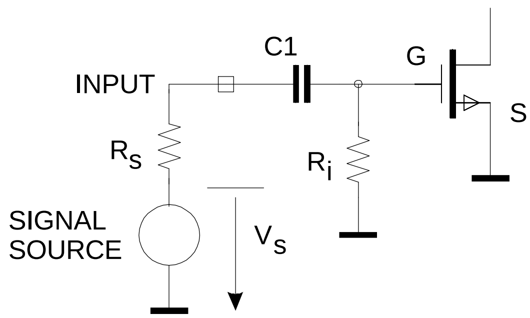
$$\frac{dI}{dt} = \frac{dI_{R1}}{dt} + \frac{dI_{R2}}{dt} = \left(\frac{1}{R1} + \frac{1}{R2}\right) \cdot \frac{dV_G}{dt} \equiv \frac{1}{R_i} \cdot \frac{dV_G}{dt}$$

where

$$R_i = \frac{R1 \cdot R2}{R1 + R2}$$

is the parallel connection of *R1* and *R2*.

Consequently, for the AC input signal the circuit is equivalent to



At the output, the voltage signal is formed by the current of the transistor flowing through the combined output load formed by R_L and $R3$.

For the moment, assume that $R_L >> R3$. Then the output load is dominated by $R3$.

The voltage at the drain D is

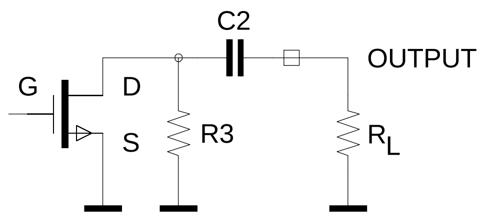
$$V_o = \mathrm{V_B} - i_D R3$$

If the gate voltage is varied, the transistor drain current changes, with a corresponding change in output voltage

$$\frac{dV_o}{di_D} = \frac{d}{dI_D}(\mathrm{V_B} - i_D R3) = R3$$

⇒ The DC supply voltage does not directly affect the signal formation.

If we remove the restriction $R_L >> R3$, the total load impedance for time-variant signals is the parallel connection of $R3$ and $(X_{C2} + R_L)$, yielding the equivalent circuit at the output



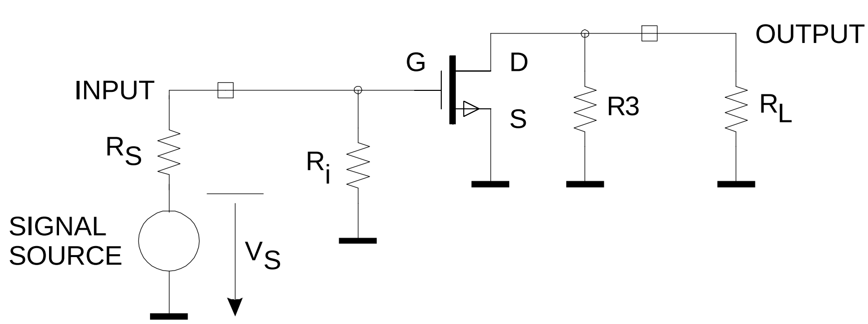
If the source resistance of the signal source $R_S << R_i$, the input coupling capacitor $C1$ and input resistance R_i form a high-pass filter. At frequencies where the capacitive reactance is $<< R_i$, i.e.

$$f >> \frac{1}{2\pi\, R_i C1}$$

the source signal v_s suffers negligible attenuation at the gate, so that

$$\frac{dV_G}{dt} = \frac{dV_s}{dt}$$

Correspondingly, at the output, if the impedance of the output coupling capacitor $C2 << R_L$, the signal across R_L is the same as across $R3$, yielding the simple equivalent circuit



Note that this circuit is only valid in the “high-pass” frequency regime.

Equivalent circuits are an invaluable tool in analyzing systems, as they remove extraneous components and show only the components and parameters essential for the problem at hand.

Often equivalent circuits are tailored to very specific questions and include simplifications that are not generally valid. Conversely, focussing on a specific question with a restricted model may be the only way to analyze a complicated situation.