

The Compact Muon Solenoid Experiment





1 December 1999

# CMS ECAL Endcap Geometry Specification

## Version 1.05

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#### Abstract

This note describes the geometry of the CMS electromagnetic endcap calorimeter in detail, to provide an unambiguous basis for engineering design and physics simulation.

## **1** Introduction

The CMS electromagnetic endcap calorimeter is a complex structure built up from a large number of "supercrystals", which are in turn built up from individual lead tungstate crystals. The crystals are arranged in a quasi-pointing geometry, with all inter-crystal gaps pointing beyond the interaction region, to avoid energy leakage. A crucial feature of the design is that the whole endcap is built using a large number of identical crystals; the crystal shape is carefully chosen to generate the desired quasi-pointing geometry.

This note gives a rather formal description of the geometry, and is intended to provide a clear and unambiguous basis for work on engineering design and detector modelling.

## 2 Definition of the crystal shape

The crystal shape is shown in fig 1. The front and rear faces are square, with sides of length  $a_1$  and  $a_2$  respectively. Two of the long sides meet in a right angle; one of these sides will always be the lower surface of the crystal, while the other is vertical. The tapering angle is determined by the construction shown in fig 2: in side view, the line connecting the end-points of the vertical sides is required to cross the plane containing the bottom surface of the crystal at a distance  $z^* + \Delta z$  from the front face, where  $z^*$  is the distance to the interaction point, and  $\Delta z$  is chosen in order to give a suitable off-pointing angle. Given  $a_2$ , the size of the rear face of the crystal, then  $a_1$ , is given by

$$a_1 = \frac{z^* + \Delta z}{z^* + \Delta z + b} a_2.$$

The crystal taper angle  $\psi$  is given by

$$\tan \psi = \frac{a_2}{z^* + \Delta z + b}.$$

At present both the crystal size and the position of the endcap in relation to the interaction region are the subject of debate. The current design is based on  $a_2 = 30$ mm,  $z^* = 3185$ mm,  $\Delta z = 1300$ mm, and the crystal length b = 220mm. This gives a taper angle of 0.3653°. The actual crystal shape is defined by rounding this angle to 0.36°, giving the crystal dimensions shown in table 1.





Figure 2: Definition of the crystal taper angle. The symbol **\*** represents the interaction point.

Dimension	Symbol	Value
Length	b	220.000mm
Rear side	$a_2$	30.000mm
Front side	$a_1$	28.618mm
Taper angle	$\psi$	<b>0.36</b> °

Table 1: Specification of endcap crystal dimensions.

## **3** Definition of the supercrystal

It is impracticable to assemble the endcaps from individual crystals. Instead, groups of 25 crystals are combined into a larger unit called a "supercrystal"; this forms the basic module for the construction.

#### 3.1 Internal structure of the supercrystal

A supercrystal consists of 25 individual crystals arranged in a square array, together with the scintillation light detectors and services. The array is physically contained in a glass-fibre alveolar structure which provides support and protection for the fragile crystals, and extends behind them to allow space for electronics and other services.

The construction of the supercrystal logically proceeds in two stages; firstly 5 individual crystals are assembled into a vertical stack; secondly, 5 such stacks are placed side-by-side to form the  $5 \times 5$  array. Both stages are described by the formalism of Appendix A.

#### 3.1.1 Creation of crystal stack

The crystal stack is constructed using equations (2) and (3) with the values

$$\gamma = \psi,$$

$$p_1 = a_1,$$

$$g_1 = 0.5 \text{mm},$$

$$\gamma = 0.$$

This allows a parallel-sided gap of 0.5mm between neighbouring crystals, which will be occupied by the internal wall of the alveolar container. Table 2 shows the height of the leading edge and the slope angle of each crystal, given that the bottom crystal rests on the plane y = 0.

Crystal	Height (mm)	Slope (deg)
1	28.618	0.000
2	57.736	0.360
3	86.856	0.720
4	115.979	1.080
5	145.106	1.440

Table 2: Details of crystal stack.



Figure 3: The 5-crystal stack: (a) side view; (b) top view.

#### 3.1.2 Assembly of crystal stacks

The crystal stack just described has a complex boundary formed by the tapering sides of the individual crystals. To simplify the construction of the supercrystal, a logical envelope is defined which is just large enough to contain the stack. The form of the envelope is shown in fig 3. Seen from above, the envelope has the same form as the single crystal, though the dimensions are slightly larger; the details are given in table 3. As before, 5 such envelopes may be placed side-by-side using equations (2) and (3), this time with the values

$$\begin{split} \gamma &= \lambda, \\ p_1 &= d_1, \\ g_1 &= 0.5 \text{mm}, \\ \gamma &= 0. \end{split}$$

The  $5 \times 5$  block thus obtained is not precisely square in cross-section; however, the width is only a few  $\mu$ m greater than the height, so that it is quite safe to allow a little extra space at the top of the block to achieve a square supercrystal. The supercrystal size is derived from this square block by three more operations:

1. extrapolating by 10mm in front of the crystals, so that the crystals are recessed into the alveolar container;

Dimension	Symbol	Value
Length	e	220.650mm
Rear width	$d_2$	30.005mm
Front width	$d_1$	28.618mm
Side Taper	$\lambda$	0.360°
Rear height	$h_2$	152.040mm
Front height	$h_1$	145.106mm
Top taper	arphi	<b>1.800</b> °

Table 3: Dimensions of the 5-crystal stack.



Figure 4: The endcap supercrystal shape.

- 2. extrapolating behind the crystals to a total length of 350mm allowing space for services and strengthening material;
- 3. adding an extra 0.25mm on each side face to allow for the outer walls of the container.

After these steps, the final supercrystal is a polyhedron of the same form as the single crystal, with the dimensions given in table 4. Fig 4 is a schematic drawing of a supercrystal.

In practice, after allowing for engineering tolerances in the various components of the supercrystal, the dimensions used in the construction of the endcap are as shown in table 5[1].

The centre of the supercrystal is defined as the mid-point of the line joining the centres of the front and rear faces.

Dimension	Symbol	Value
Length	В	350.000mm
Rear side	$A_2$	156.296mm
Front side	$A_1$	145.291mm
Taper angle	$\Psi$	<b>1.801</b> °

Table 4: Calculated endcap supercrystal dimensions.

Dimension	Symbol	Value
Length	В	350.000mm
Rear side	$A_2$	156.504mm
Front side	$A_1$	145.500mm
Taper angle	$\Psi$	1.801°

S/Crystal	Height (mm)	Slope (deg)
1	145.900	0.000
2	292.472	1.801
3	439.262	3.601
4	586.416	5.402
5	734.081	7.203
6	882.409	9.004
7	1031.553	10.804
8	1181.671	12.605
9	1332.927	14.406
10	1485.489	16.206
11	1639.535	18.007

Table 5: Working dimensions of endcap supercrystal.

Table 6: Details of supercrystal standard column.

## 4 Assembly of supercrystals into endcap

#### 4.1 Stacking of supercrystals into columns

The construction of the endcap is analogous to the arrangement of individual crystals to form a supercrystal. The endcap is assembled conceptually, if not physically, from columns of supercrystals of various heights. All of the real columns are derived from a standard column of eleven supercrystals by truncation at the top or bottom to achieve an approximately circular profile, as explained in more detail below.

The standard column is built on a base supercrystal with its lower face in the horizontal (x, z) plane, in the same way as the crystal stack described in section 3.1.2. In the notation of Appendix A,

$$\sigma = \Psi,$$
  

$$p_1 = A_1,$$
  

$$g_1 = 1.0$$
mm,  

$$g_2 = 1.0$$
mm.

Given that the base supercrystal has a slope angle  $\Theta = 0$ , and its lower face is at a distance of 0.4mm from the (x, z) plane, equations (2) and (3) completely specify the structure of the standard column.

Table 6 shows the height of the leading edge and the slope angle for the 10 supercrystals in the standard column.

#### 4.1.1 Definition of the column envelope

The supercrystal column has a complicated boundary formed by the tapering sides of the individual supercrystals.



Figure 5: Dimensions of column envelope; (a) side view, (b) top view

As before, a more regular envelope is defined of the minimal size needed to contain all of the supercrystals of a column. Fig 5 shows the shape of this envelope. Seen from above, the envelope has the same trapezoidal profile as the supercrystals; however, the depth E and the tapering angle  $\Lambda$  differ from the corresponding supercrystal measurements.

The angle  $\Lambda$  is determined by the requirement that the envelope must taper enough to contain the topmost supercrystal, sloping at an angle  $\Theta^{\text{top}}$  (note that for the purposes of this calculation, supercrystals which are cut away to tailor the endcap to a circular profile are treated as whole supercrystals); thus for column *j*,

$$\tan \Lambda_i = (A_2 - A_1) \cos \Psi / B \cos (\Theta^{\text{top}}_i + \Psi).$$

It should be noted that this angle depends on the height of the column.

The envelope depth is defined by the supercrystal which projects furthest back from the plane containing the leading edges. For the columns used in the endcap, this is always the topmost supercrystal, giving

$$E = B\cos\Theta^{\mathbf{top}} + A_1\sin\Theta^{\mathbf{top}}$$

Column	Range	X(cm)	Φ(°)
1	3-11	146.500	0.000
2	2-11	293.082	1.913
3	1-10	439.909	3.826
4	1-10	587.142	5.718
5	1-10	734.944	7.611
6	1-9	883.482	9.503
7	1-9	1032.919	11.378
8	1-8	1183.427	13.252
9	1-7	1335.173	15.110
10	1-5	1488.329	16.954
11	1-2	1643.064	18.777

Table 7: Specification of supercrystal columns in endcap. The position X is the x-coordinate of the vertical edge of the column furthest from the vertical (y) axis, while  $\Phi$  is the rotation angle with respect to the (y, z) plane.

The remaining distances in fig 5 are now trivially calculated.

$$D_1 = A_1$$
$$D_2 = D_1 + E \tan \Lambda$$
$$F = E / \cos \Lambda$$
$$H_2 = H_1 + E \tan \left(\Theta^{top} + \Psi\right)$$

where the height  $H_1$  of the front of the column is derived from the stacking procedure defined in appendix A.

#### 4.2 Stacking of columns

#### 4.2.1 Construction of a quadrant

The next stage in the endcap construction is to stack columns next to each other to make a quadrant. Table 7 defines the parameters of each of the columns. Some of the supercrystals may be cut away so that the overall shape of the endcap approximates a circle, though the geometry is calculated as if these cut-away supercrystals were complete. Column 1 is offset from the *y*-axis by a distance of 1mm to allow clearance for the physical assembly of the quadrant from two mirror-image "Dees".

In this procedure, the boundary of the column is the envelope defined in section 4.1.1. The geometry is again that of appendix A. In this case, the inter-column gap  $\ell$  is assumed to be 0.5mm at the front and the rear, so there is no analogue of the angle  $\gamma$ . Column (j + 1) is positioned relative to column j by:

- 1. rotation to an angle  $\Phi_{j+1} = \Phi_j + \Lambda_j$ , where the axis of rotation is the leading edge of the column, and the first column (nearest to the *Y* axis) has  $\Phi_1 = 0$ ;
- 2. translation of the rotated column in the positive X direction through a distance  $u_{j+1}$  given by

$$u_{j+1} = (D_1 + \ell) / \cos \Phi_{j+1}.$$

### 5 Summary and Conclusions

The preceding sections have provided a detailed description of the CMS electromagnetic endcap calorimeter. The detector is built up in steps starting from individual crystal; in each step, a set of trapezia are stacked together to form a larger module. The geometry of such stacks is analysed in appendix A. The construction of the physical detector naturally requires support structures and services which have been ignored here, but this note should explain the layout of the active components of the detector, and the detector model implemented in the CMS detector simulation software.

## 6 Acknowledgements

Thanks are due to Jon Hays (Imperial College) and Len Denton (RAL) for their work in making independent calculations of the endcap geometry, and providing essential cross-checks on the calculations presented here.

## A Geometrical calculations

The geometry of the endcap, and the interior structure of the supercrystals, arise through the stacking of trapezia as illustrated in fig 6. These trapezia have the property that two of the interior angles are right-angles. This appendix sets out the geometry of such stacks in detail.

Consider a trapezium, with the dimensions defined in fig 6(a), sloping at an angle  $\sigma_1$  with respect to the *X* axis. A column is to be formed with the leading corners of all members of the column positioned on a line at constant *X*. The position of the next trapezium in the column may be obtained by rotating the original about the leading corner to angle  $\sigma_2$  and translating by a distance *t* in the *Y* direction, to obtain the position shown in fig 6(b).

The trapezia are separated by a gap which, for maximum generality, need not be parallel-sided. The gap is defined by the widths  $g_1$  at the front of the upper trapezium, and  $g_2$  at the rear of the lower trapezium, with both distances normal to the lower face of the upper trapezium.

A number of geometric relations follow at once from fig 6. Defining the gap angle  $\gamma$  by

$$\sin \gamma = (g_2 - g_1)/(r - s) \equiv \Delta g/(r - s), \tag{1}$$

then

$$\sigma_2 = \sigma_1 + \tau + \gamma, \tag{2}$$

$$t = (p_1 + g_1) \cos \gamma / \cos \left(\sigma_1 + \tau\right), \tag{3}$$

$$s = (p_1 + g_1) \sin \sigma_2 / \cos (\sigma_1 + \tau).$$
 (4)

In order to calculate  $\sigma_2$  and t,  $\gamma$  must be evaluated. Making the approximation that  $\cos \gamma = 1$  (a very good approximation in the endcap geometry), (4) becomes

$$s = (p_1 + g_1)(\tan(\sigma_1 + \tau) + \sin\gamma).$$
 (5)

Then, substituting (5) into (1),

$$\sin \gamma = \Delta g / (r' - (p_1 + g_1) \sin \gamma),$$

where

$$r' = r - (p_1 + g_1) \tan(\sigma_1 + \tau).$$

Thus,  $\sin \gamma$  is obtained by solving

$$(p_1 + g_1)(\sin\gamma)^2 - r'\sin\gamma + \Delta g = 0$$
(6)



Figure 6: Stacking geometry of trapezia: (a) trapezium dimensions, (b) stacking parameters.

giving

$$\sin \gamma = \frac{r'}{2(p_1 + g_1)} (1 - \sqrt{1 - 4\Delta g(p_1 + g_1)/{r'}^2})$$
$$\sin \gamma = \Delta g/r'$$
(7)

or

provided that  $r'^2 \gg 4\Delta g(p_1 + g_1)$ . Note that  $\gamma$  is not constant, but rather is a function of  $\sigma_1$ .

## References

[1] B Smith, private communication