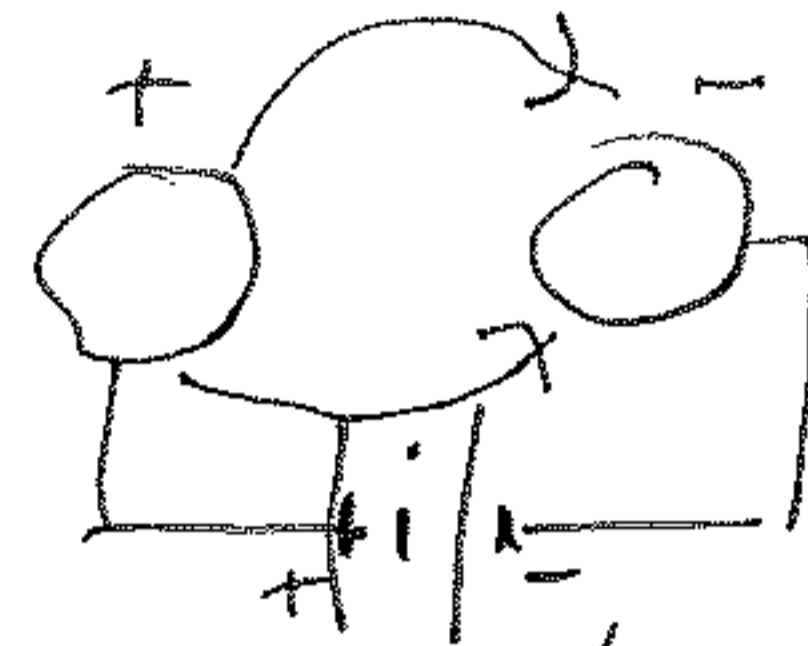


FEB 24

The capacitance of a capacitor is a physical property of the 2-conductor system; It depends on geometry and permittivity of the medium between them; A capacitor has a capacitance even when no voltage is applied



To find C :

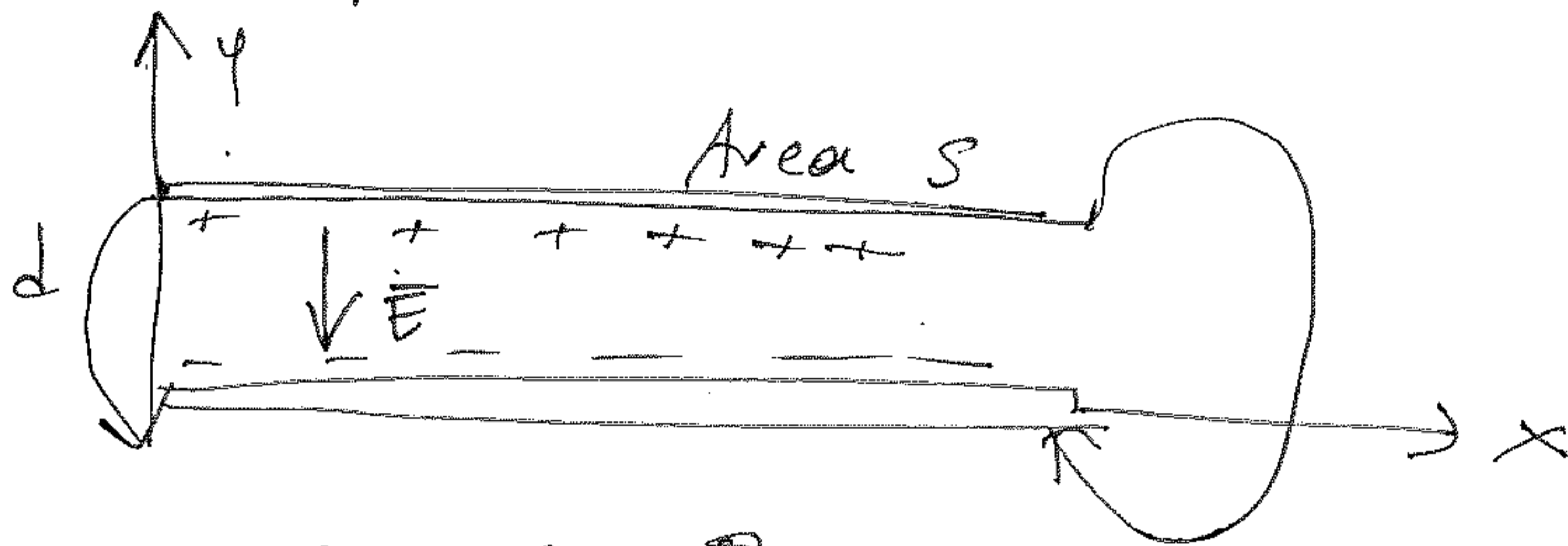
1. Choose appropriate coordinate system
2. Assume $+Q$ and $-Q$ on the conductors
3. Find \vec{E} from Q by

$$\boxed{E_{in} = \frac{\rho_s}{\epsilon_1}}$$
 Gauss' law
4. Find V_{12} by evaluating

$$V_{12} = -\int_2^1 \vec{E} \cdot d\vec{l}$$
5. Find C by taking the ratio $\frac{Q}{V_{12}}$



A parallel-plate capacitor consists of two parallel conducting plates of area S separated by a uniform distance d . The space is filled with dielectric of constant permittivity ϵ ; What is the capacitance



$$+ \rho_s, - \rho_s \quad \rho_s = \frac{Q}{S}$$

$$\vec{E} = \frac{\rho_s}{\epsilon} (-\hat{a}_y) = -\hat{a}_y \frac{Q}{\epsilon S}$$

$$V_{12} = - \int_{y=0}^{y=d} \vec{E} \cdot d\vec{l} = - \int_0^d (-\hat{a}_y) \frac{Q}{\epsilon S} \cdot (\hat{a}_y dy)$$

$$= \frac{Q}{\epsilon S} d$$

$$C = \frac{Q}{V_{12}} = \epsilon \frac{S}{d}$$

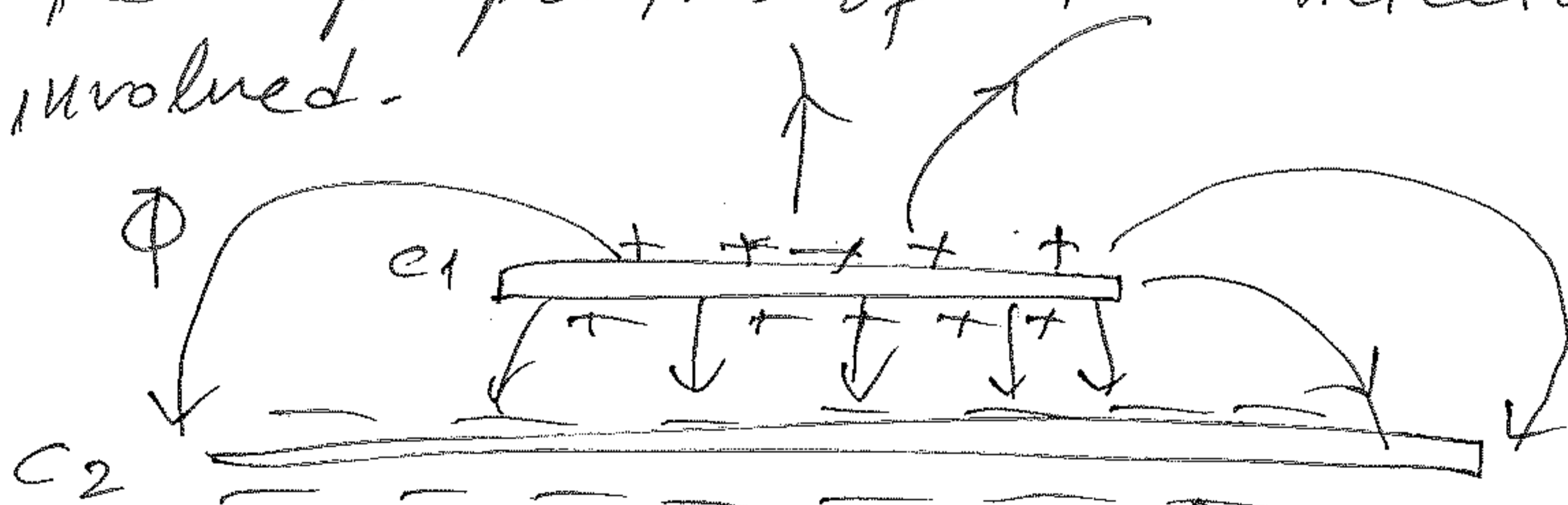
CAPACITANCE

Any two conducting bodies separated by free space or a dielectric material have a capacitance between them.

A voltage difference applied results in a charge $+Q$ on one conductor and $-Q$ on the other. The ratio of the absolute value of the charge to the absolute value of the voltage difference is defined as the capacitance of the system:

$$C = \frac{Q}{V} \text{ (F)} \quad 1\text{F} = \frac{1\text{C}}{1\text{V}}$$

The capacitance depends only on the geometry of the system and the properties of the dielectrics involved.



$+Q$ is placed on C1 (conductor 1) and $-Q$ on C2

the flux of the electric field is as shown

Parallel plate capacitor

$$C = \frac{Q}{V_{12}} = \epsilon \frac{S}{d}$$

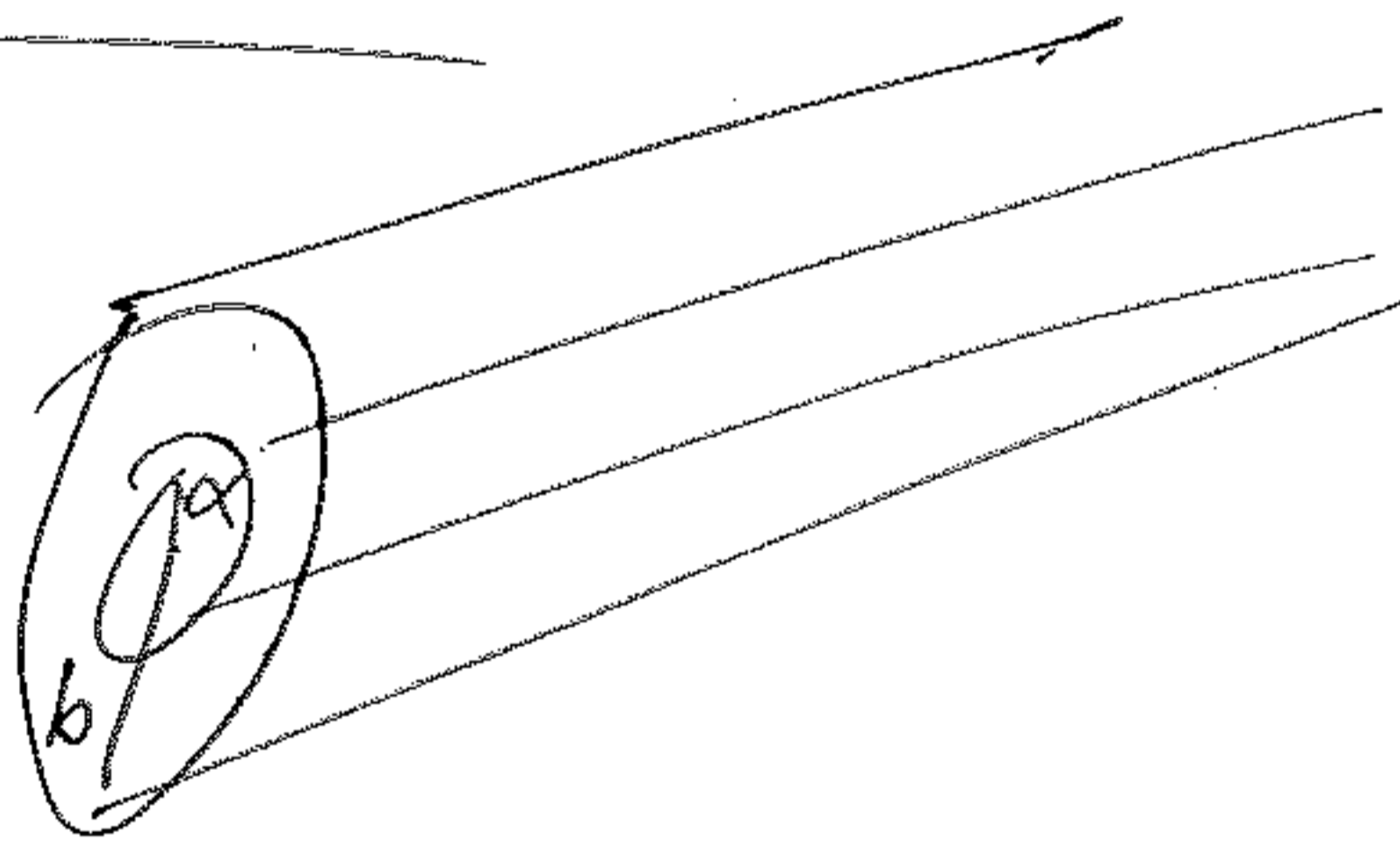
independent of ϵ or V_{12}

2) Cylindrical capacitor

A cylindrical capacitor consists of inner conductor of radius a and outer of radius b . The space is filled with dielectric of permittivity ϵ and the length of the capacitor is L .

Capacitance?

→ Use cylindrical



Assume charges $+Q$, $-Q$ on the surface of the inner conductor and the inner surface of the outer conductor

derive \vec{E} Gauss' law in $a < r < b$.

$$\vec{E} = \hat{r} E_r = \hat{r} \frac{Q}{2\pi\epsilon L r}$$



$$V_{ab} = - \int_{r=b}^{r=a} \vec{E} \cdot d\vec{l} = - \int_b^a \left(\hat{a}_r \frac{Q}{2\pi\epsilon L r} \right) \cdot \left(\hat{a}_r dr \right)$$

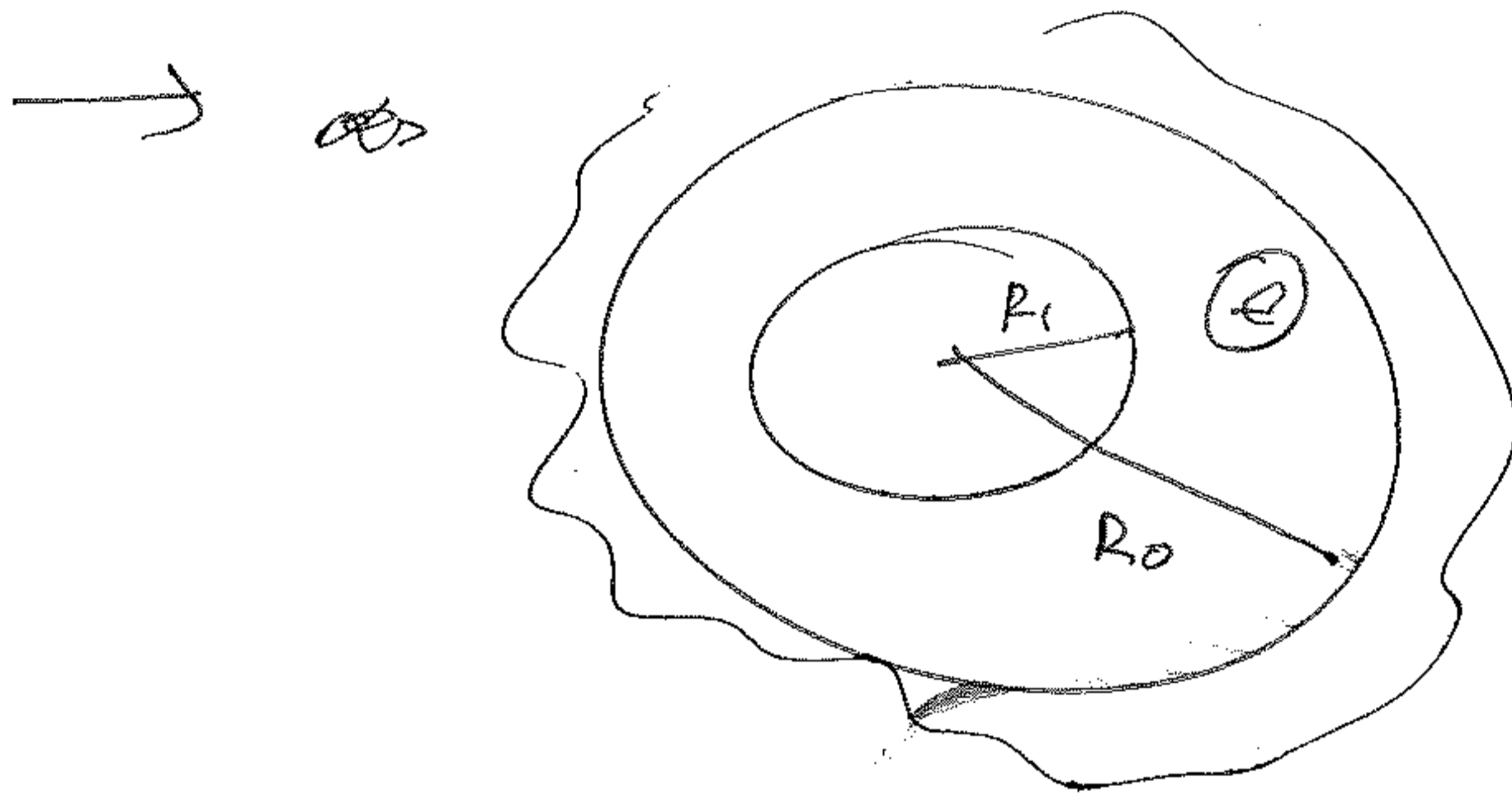
$$= \frac{Q}{2\pi\epsilon L} \ln \frac{b}{a}$$

Cylindrical capacitor

$$C = \frac{Q}{V_{ab}} = \frac{2\pi\epsilon L}{\ln \frac{b}{a}}$$



A spherical capacitor consists of an inner conducting sphere of radius R_i and an outer conductor with a spherical inner wall of radius R_o . Space in between permittivity ϵ . What is the capacitance?



Assume $+Q$, $-Q$ on the inner and outer conductors of the spherical capacitor. By Gauss' law $R (R_i < R < R_o)$ we have

$$\vec{E} = \hat{a}_r E_r = \hat{a}_r \frac{Q}{4\pi\epsilon R^2}$$

$$V = - \int_{R_o}^{R_i} \vec{E} \cdot (\hat{a}_r dr) =$$

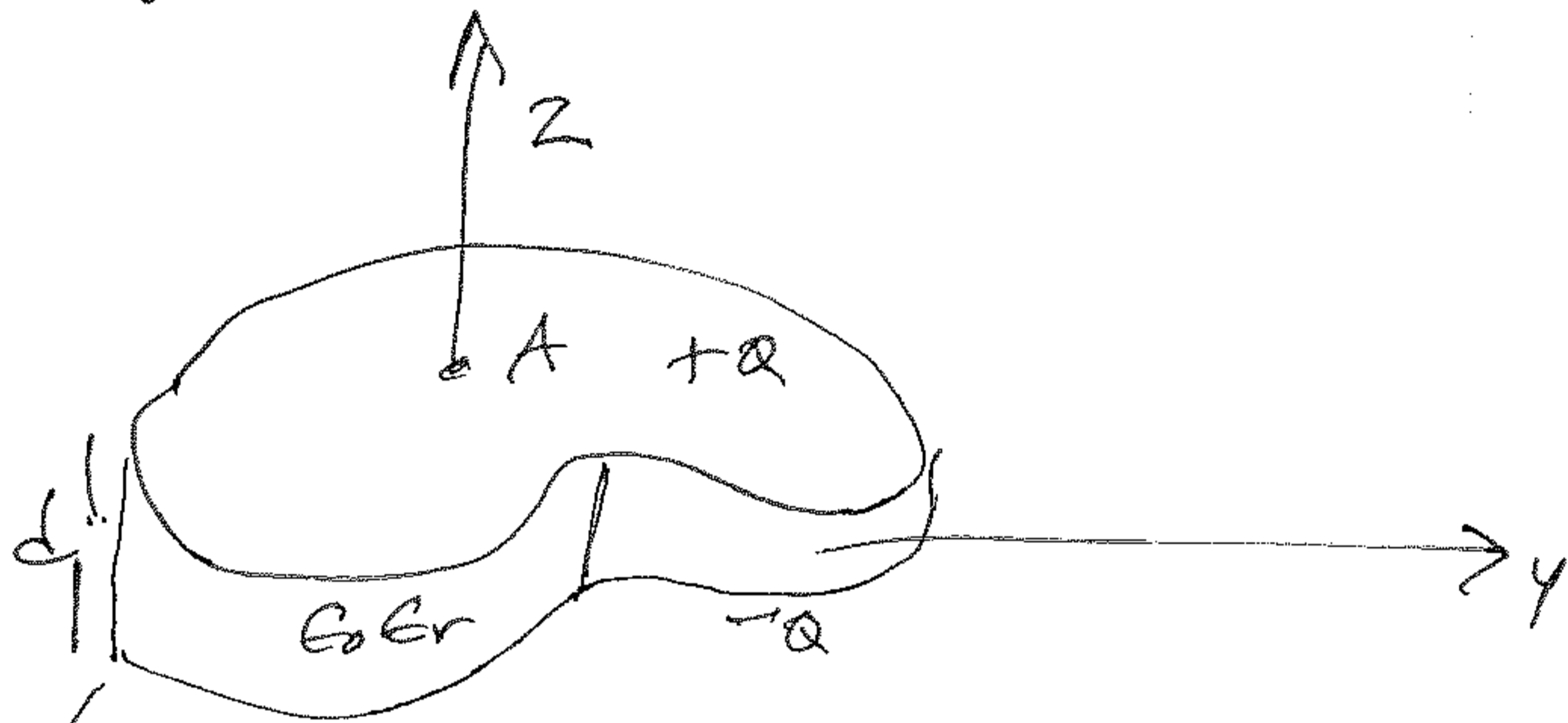
$$= - \int_{R_o}^{R_i} \frac{Q}{4\pi\epsilon R^2} dR = \frac{Q}{4\pi\epsilon} \left(\frac{1}{R_i} - \frac{1}{R_o} \right)$$

Spherical capacitor

$$C = \frac{Q}{V} = \frac{4\pi\epsilon}{\frac{1}{R_i} - \frac{1}{R_o}}$$



Flux \rightarrow I have \vec{E} and \vec{D}
 To ~~be~~ double the charges would simply
 double \vec{E} and \vec{D} ~~but~~ \Rightarrow
 double the $V \Rightarrow C = Q/V$
 ($V = -\int \vec{E} \cdot d\vec{l}$)



Find the capacitance neglecting fringing

with $+Q$ on top and $-Q$ on the bottom

$$\rho_s = \frac{Q}{A}$$

$$\boxed{E_{\perp} = \frac{\rho_s}{\epsilon_0}}$$

$$D_{\perp} = \rho_s = \frac{Q}{A}$$

$$\vec{D} = \frac{Q}{A} (-\hat{x}_2)$$

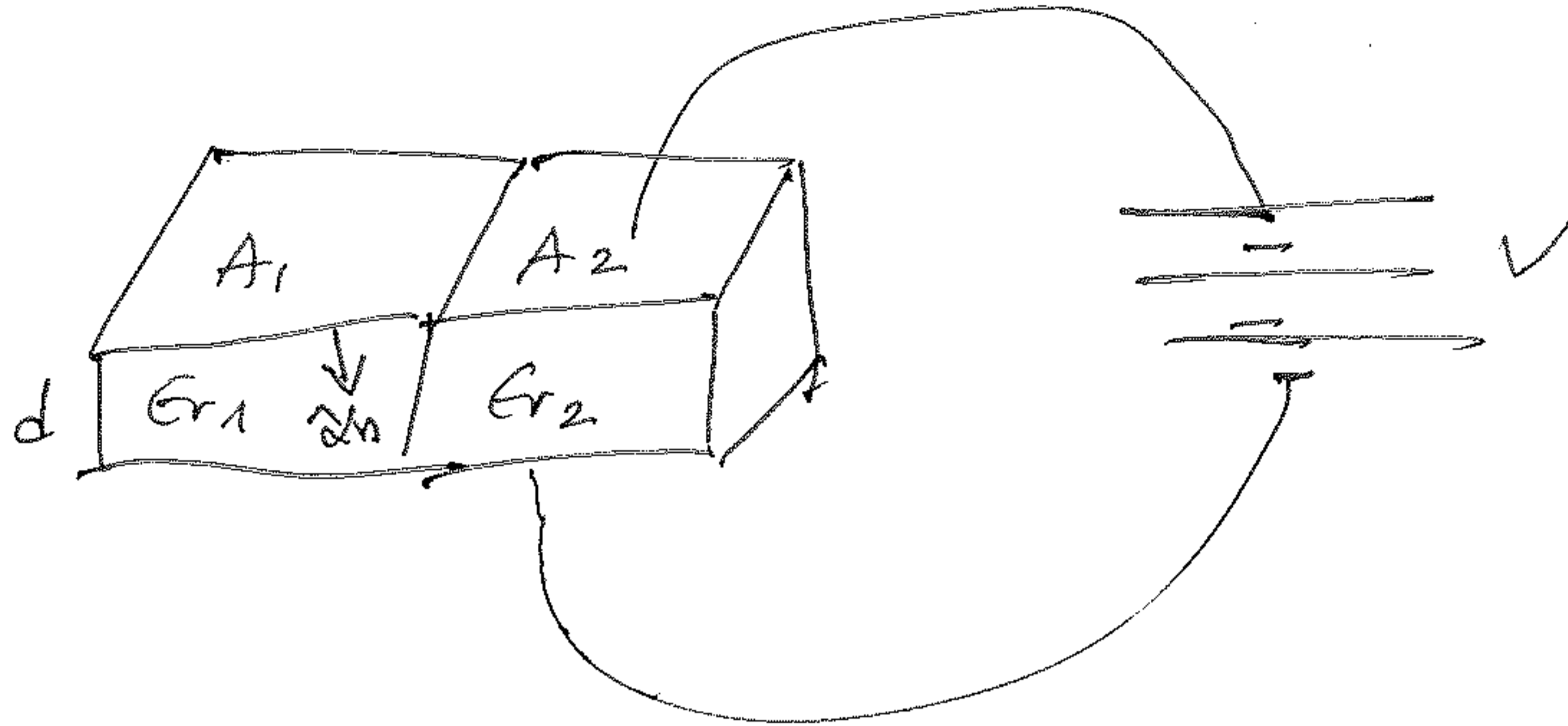
$$\vec{E} \Rightarrow \frac{Q}{A \epsilon_0 \epsilon_r} (-\hat{x}_2)$$

$$= \epsilon_0 \epsilon_r \vec{E}$$

$$V = - \int_0^d \frac{Q}{\epsilon_0 \epsilon_r A} (-\hat{x}_2) \cdot dz \hat{x}_2 =$$

$$= \frac{Qd}{\epsilon_0 \epsilon_r A}$$

$$\boxed{C = \frac{Q}{V} = \frac{\epsilon_0 \epsilon_r A}{d}}$$



show that $C_{eq} = \frac{\epsilon_0 \epsilon_{r1} A_1}{d} + \frac{\epsilon_0 \epsilon_{r2} A_2}{d} = C_1 + C_2$

Side-by-side dielectrics: the equivalent capacitance is the sum of the individual capacitances.

V is common.

$$\vec{E}_1 = \vec{E}_2 = \frac{V}{d} \hat{a}_n$$

$$\vec{D} = \epsilon_0 \epsilon_r \vec{E} \Rightarrow \vec{E} = \frac{\vec{D}}{\epsilon_0 \epsilon_r}$$

$$\frac{\vec{D}_1}{\epsilon_0 \epsilon_{r1}} = \frac{\vec{D}_2}{\epsilon_0 \epsilon_{r2}} = \frac{V}{d} \hat{a}_n$$

\hat{a}_n is the downward normal to the upper plate

$$D_n = \rho_s$$

$$\rho_{s1} = \frac{V}{d} \epsilon_0 \epsilon_{r1}$$

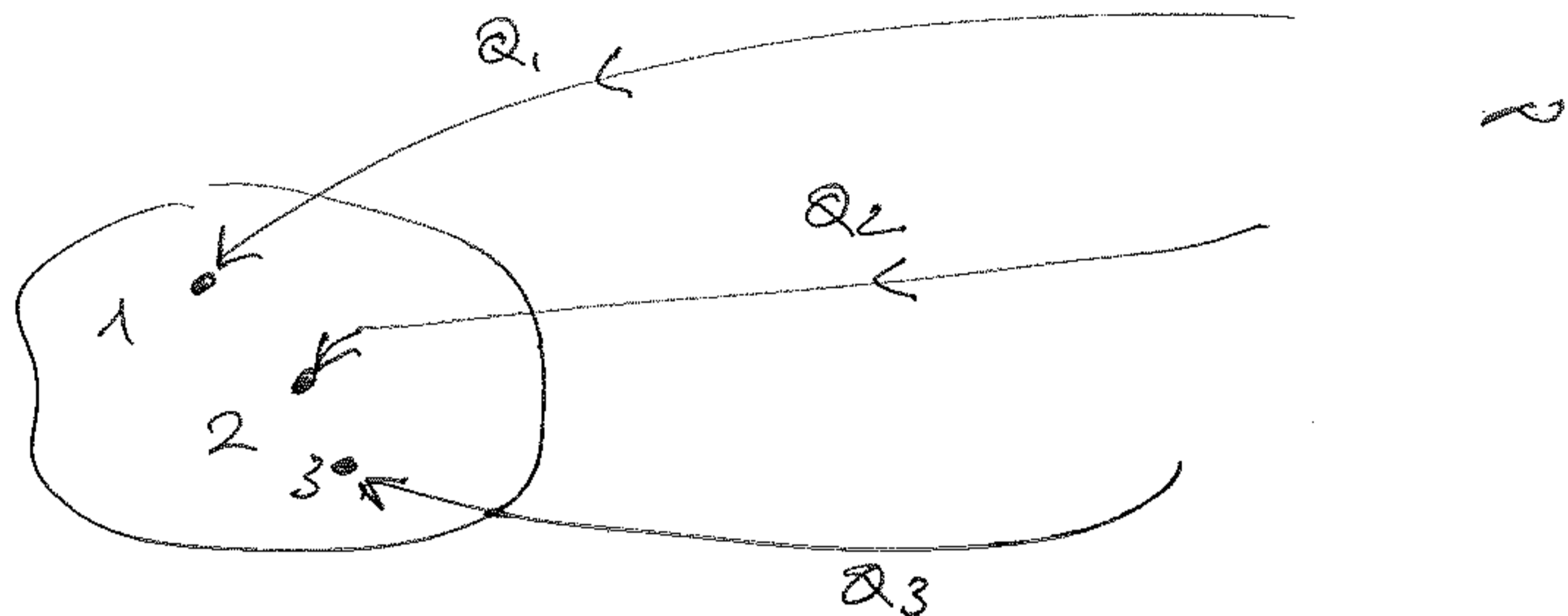
$$\rho_{s2} = \frac{V}{d} \epsilon_0 \epsilon_{r2}$$

$$Q = \rho_{s1} \cdot A_1 + \rho_{s2} \cdot A_2 = V \left(\frac{\epsilon_0 \epsilon_{r1} A_1}{d} + \frac{\epsilon_0 \epsilon_{r2} A_2}{d} \right)$$

$$C_{eq} = \frac{Q}{V} = C_1 + C_2$$

$$E_1 = \frac{\rho_s}{\epsilon_0} = \frac{V}{d}$$

Energy in static E field



Consider the work done to assemble charge by charge a distribution $n=3$ point charges. The region is assumed initially to be charge free and with $\vec{E} = 0$ throughout.

to bring $Q_1 \rightarrow 1$ $W_1 = 0$ (no \vec{E})

to bring $Q_2 \rightarrow 2$ $W_2 = Q_2 \cdot V_{2,1}$

the potential @ point 2 due to charge 1

to bring $Q_3 \rightarrow 3$ $W_3 = Q_3 V_{3,1} + Q_3 V_{3,2}$

If the charges were put in reverse order $\overline{W_E} = 0 + W_2 + W_3 = 0 + Q_2 V_{2,1} + Q_3 V_{3,1} + Q_3 V_{3,2}$

W_E = $W_3 + Q_2 V_{3,2} + Q_1 V_{1,3} + Q_1 V_{1,2}$

|| ||

0 W_2

$2W_E = \underbrace{Q_1 (V_{1,2} + V_{1,3})}_{Q_1 \cdot V_1} + \underbrace{Q_2 (V_{2,1} + V_{2,3})}_{V_2} + \underbrace{Q_3 (V_{3,1} + V_{3,2})}_{V_3}$

V_i is the potential @ P_i $2W_E = Q_1 V_1 + Q_2 V_2 + Q_3 V_3$



$$W_E = \frac{1}{2} (Q_1 V_1 + Q_2 V_2 + Q_3 V_3) = \frac{1}{2} \sum_{m=1}^n Q_m V_m$$

for a region containing n point charges

for a region w/ charge density ρ (C/m³)

the summation becomes integration

$$W_E = \frac{1}{2} \int \rho V dV$$

Other forms

$$\left. \begin{aligned} W_E &= \frac{1}{2} \int \vec{D} \cdot \vec{E} dV \\ W_E &= \frac{1}{2} \int \epsilon E^2 dV \\ W_E &= \frac{1}{2} \int \frac{D^2}{\epsilon} dV \end{aligned} \right\} \text{same}$$

In an electric circuit the energy stored in the field of a capacitor is given by

$$W_E = \frac{1}{2} QV = \frac{1}{2} CV^2$$

where C is the capacitance in Farads
 and V is the voltage difference between 2 conductors making up the capacitor and Q is the magnitude of total charge in one of the conductors



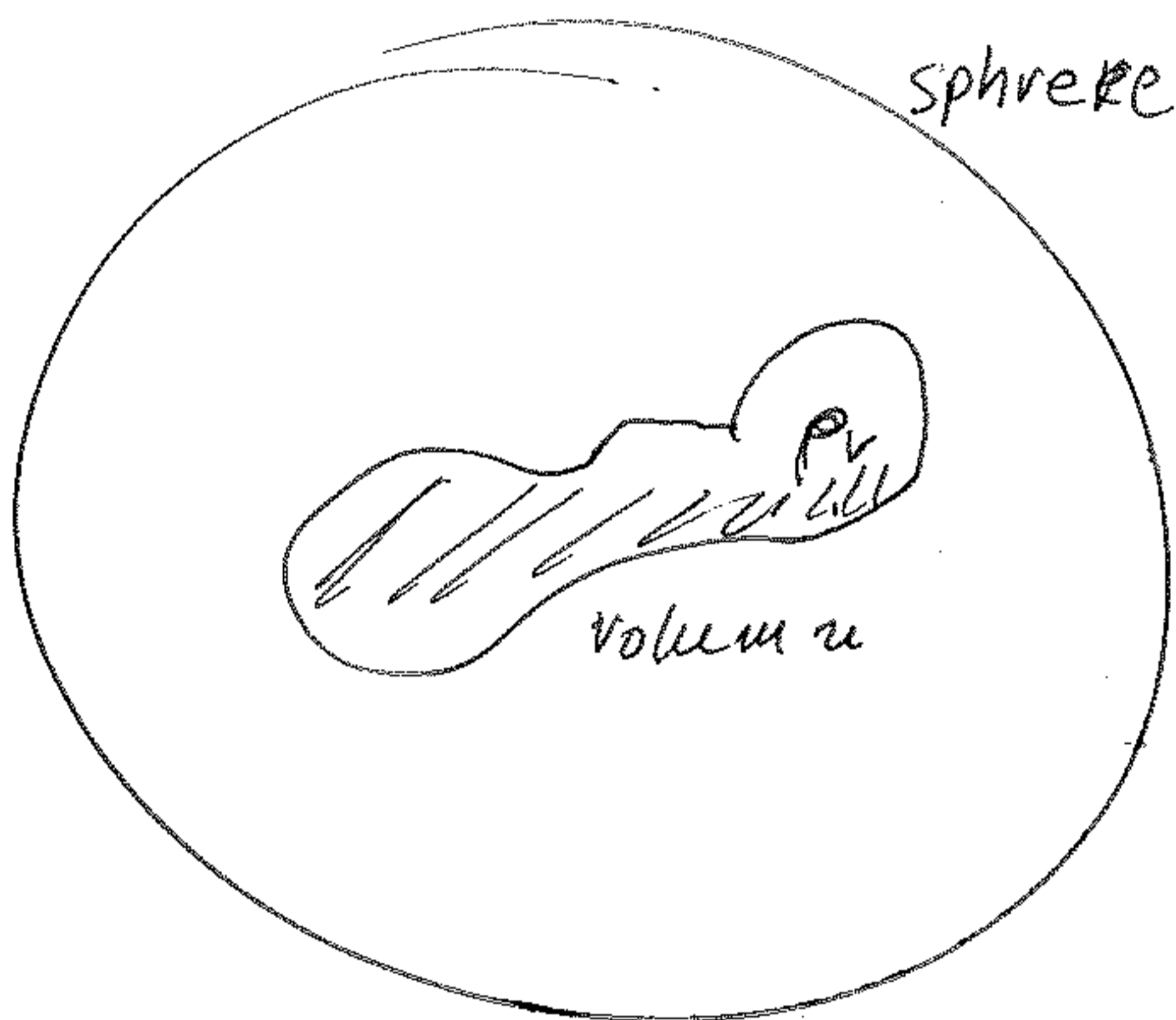
$$\epsilon \bar{E} = \bar{D}$$

Prove that

$$W_E = \frac{1}{2} \int \rho E^2 du = \frac{1}{2} \int \bar{D} \cdot \bar{E} du \quad (11)$$

Charge distributed through a volume u with density ρ gives rise to an electric field

$$\text{with } W_E = \frac{1}{2} \int_u \rho \phi du \quad \text{or} \quad W_E = \frac{1}{2} \int \epsilon E^2 du$$



$$\oint \bar{D} \cdot d\bar{S} = \int \rho du = Q_{enc}$$

$$\rho = \nabla \cdot \bar{D}$$

$$\oint \bar{D} \cdot d\bar{S} = \int (\nabla \cdot \bar{D}) du$$

charge-containing volume u enclosed within
 large sphere R , $\rho = 0$ outside u

$$W_E = \frac{1}{2} \int_u \rho \phi du = \frac{1}{2} \int_{\text{spherical volume}} \rho \phi du = \frac{1}{2} \int_{\text{spherical volume}} (\nabla \cdot \bar{D}) \phi du$$

$$\nabla \cdot \phi \bar{A} = \bar{A} \cdot \nabla \phi + \phi (\nabla \cdot \bar{A}) \Rightarrow$$

$$\Rightarrow \phi (\nabla \cdot \bar{A}) = \nabla \cdot \phi \bar{A} - \bar{A} \cdot \nabla \phi$$

$$= \frac{1}{2} \int \nabla \cdot \phi \bar{D} du - \frac{1}{2} \int (\bar{A} \cdot \nabla \phi) du$$



$$W_{\vec{v}} = \frac{1}{2} \int_{\text{sphere}} (\vec{\nabla} \cdot \nabla \vec{D}) du - \frac{1}{2} \int_{\text{sphere}} (\vec{D} \cdot \vec{\nabla} v) du$$

take $R \rightarrow \infty$

divergence theorem

$$\oint_{\text{surface}} v \vec{D} \cdot d\vec{S}$$

take $R \rightarrow \infty$ then the enclosed volume looks like a point charge. At the surface

\vec{D}	appears $\sim \frac{K_1}{R^2}$	} integrant $\frac{1}{R^3}$
v	appears $\sim \frac{K_2}{R}$	

$$\lim_{R \rightarrow \infty} \left(\oint_{\text{surface}} v \vec{D} \cdot d\vec{S} = \right) \quad dS \rightarrow R^2$$

$$\lim_{R \rightarrow \infty} \left(\frac{R^2}{R^3} \right) = 0$$



$$\vec{D} = \epsilon \vec{E}$$

$$\vec{E} = -\nabla V$$

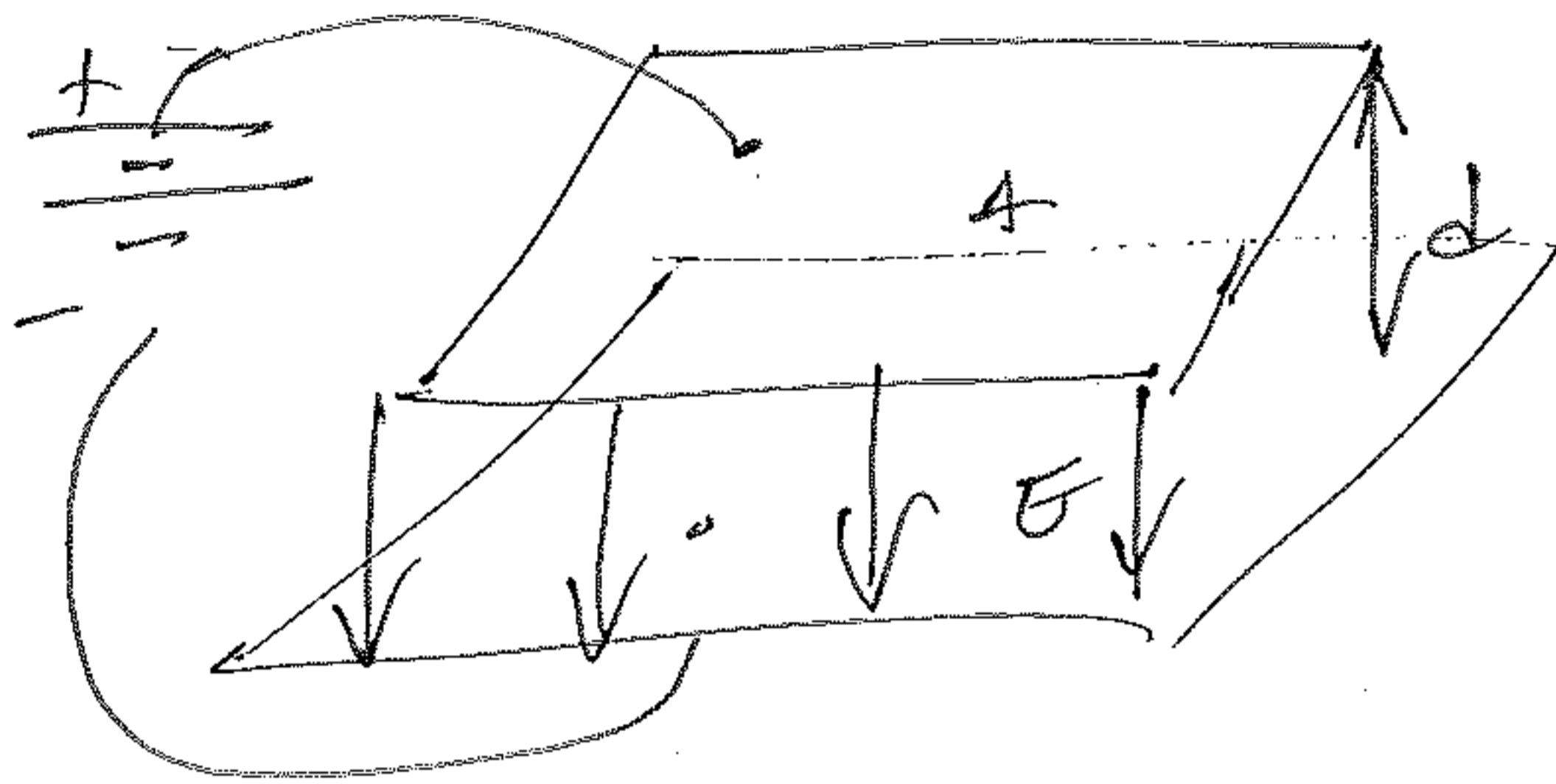
$$-\frac{1}{2} \int (\vec{D} \cdot \vec{\nabla} V) du =$$

(13)

$$= \frac{1}{2} \int_{\text{sphere}} \epsilon \vec{E} \cdot (-\vec{E}) du = + \frac{1}{2} \int_{\text{spherical volume}} \epsilon \vec{E}^2 du$$

$$W_E = 0 + \frac{1}{2} \int \epsilon \vec{E}^2 du = \frac{1}{2} \left(\int \frac{\vec{D}^2}{\epsilon} du \right)$$





$$E = \frac{\rho}{2\epsilon_0}$$

$$\rho = \frac{dq_s}{ds}$$

A parallel plate capacitor for which $C = \epsilon \frac{A}{d}$ has a $V = ct$ across the plates.

Find the stored energy in the electric field.

→ Fringing neglected (edge effects)

$$\vec{E} = \left(\frac{V}{d} \right) \hat{a}_n \quad \text{between the plates}$$

$$= \left(\frac{V}{d} \right) \hat{a}_n \quad \text{and } E = 0 \text{ elsewhere.}$$

$$dV = -\vec{E} \cdot d\vec{l}$$

$$\vec{E} = -\nabla V$$

$$W_e = \frac{1}{2} \int_{\text{volu}} \epsilon E^2 d\tau = \frac{\epsilon}{2} \left(\frac{V}{d} \right)^2 \int_{\text{volu}} d\tau$$

$$= \frac{\epsilon}{2} \left(\frac{V}{d} \right)^2 A \cdot d$$

$$= \frac{\epsilon V^2 A}{2d} = \frac{1}{2} CV^2$$

Second method

The total charge on one conductor maybe found from $\vec{D} = \epsilon \vec{E}$ at the surface via Gauss's law

$$\vec{D} = \epsilon \frac{V}{d} \hat{a}_n$$

$$Q = |\vec{D}| \cdot A$$

$$= \epsilon \frac{V}{d} \hat{a}_n \cdot A$$

$$= \frac{\epsilon V A}{d}$$

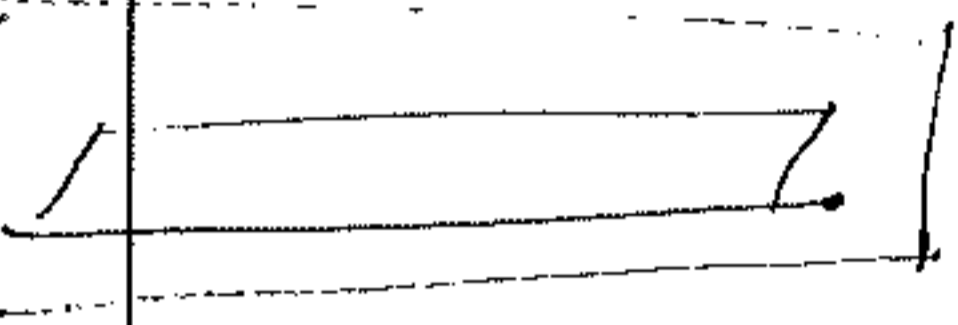
$$\oint \vec{D} \cdot d\vec{S} = \int \rho dV$$

$$= Q_{enc}$$

$$\oint \epsilon \vec{E} \cdot d\vec{S} = \int \rho dV$$

$$= Q_{enc}$$

No. 5505
Engineer's Computation Pad
AMIN



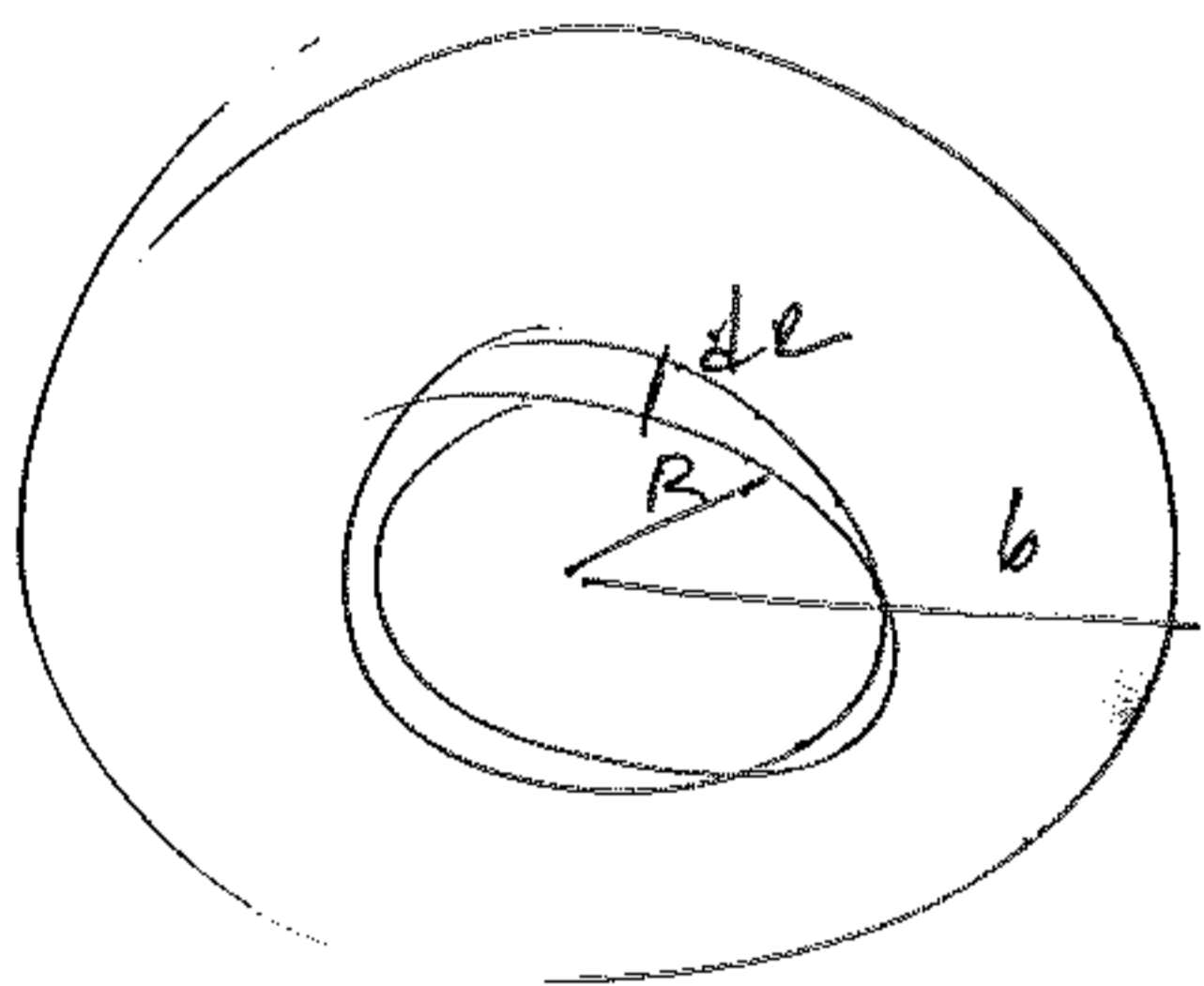
$$W = \frac{1}{2} QV = \frac{1}{2} \frac{\epsilon V A}{d} V = \frac{1}{2} \frac{\epsilon A V^2}{d} = \frac{1}{2} C V^2$$

Potential energy of a group of N discrete
~~two~~ point charges N at rest

$$W = \frac{1}{2} \sum_{k=1}^N Q_k V_k$$

$$V_k = \frac{1}{4\pi\epsilon_0} \sum_{\substack{j=1 \\ j \neq k}}^N \frac{Q_j}{R_{jk}}$$

Find the energy required to assemble
a uniform sphere of charge of radius
 b and volume charge density ρ



Because of symmetry
simple to assume that
the sphere

$$V_R = \frac{Q_R}{4\pi\epsilon_0 R}$$

$$Q_R = \rho \frac{4}{3}\pi R^3$$

$$dQ_R = \rho 4\pi R^2 dR$$

$$dW = V_R dQ_R = \frac{4\pi}{3\epsilon_0} \rho^2 R^4 dR$$

Total work to assemble a uniform sphere
of charge of radius b and charge density
 ρ is

$$W = \int dW = \frac{4\pi}{3\epsilon_0} \rho^2 \int_0^b R^4 dR = \frac{4\pi \rho^2 b^5}{15\epsilon_0} \quad (5)$$

$$\vec{D} = \epsilon \vec{E}$$

In terms of the total charge

$$Q = \rho \frac{4\pi}{3} b^3$$
$$W = \frac{3Q^2}{20\pi\epsilon_0 b}$$

Energy is directly proportional to the square of the total charge and inverse proportional to the radius (could be a cloud of electrons)

for a continuous charge distribution of density ρ $Q_R \rightarrow \rho dV$

$$W = \frac{1}{2} \int_V \rho V dV$$

Solve the same prob using this \rightarrow

$$W = \frac{1}{2} \rho \int_V V dV = \frac{\rho}{2} \int_0^b V 4\pi R^2 dR$$

V is the potential at point R

$$\vec{E} \begin{cases} \vec{E}_1 = \hat{a}_R E_{R1} & R = \infty \text{ to } b \\ \vec{E}_2 = \hat{a}_R E_{R2} & R = b \text{ to } 0 \end{cases}$$

$$\vec{E}_{R1} = \hat{a}_R \frac{Q}{4\pi\epsilon_0 R^2} = \hat{a}_R \frac{\rho b^3}{3\epsilon_0 R^2} \quad R > b$$

$$\vec{E}_{R2} = \hat{a}_R \frac{Q_R}{4\pi\epsilon_0 R^2} = \hat{a}_R \frac{\rho R}{3\epsilon_0} \quad 0 \leq R \leq b$$

$$\boxed{\vec{D} = \epsilon \vec{E}}$$

$$V = - \int_{\infty}^R \vec{E} \cdot d\vec{R} =$$

$$= - \left[\int_{\infty}^b E_{R1} dR + \int_0^R E_{R2} dR \right]$$

$$= - \left[\int_{\infty}^b \frac{\rho b^3}{3\epsilon_0 R^2} dR + \int_0^R \frac{\rho R}{3\epsilon_0} dR \right]$$

$$= \frac{\rho}{3\epsilon_0} \left(b^2 + \frac{b^2}{2} - \frac{R^2}{2} \right) = \frac{\rho}{3\epsilon_0} \left(\frac{3}{2} b^2 - \frac{R^2}{2} \right)$$

$$W = \frac{\rho}{2} \int_0^b \frac{\rho}{3\epsilon_0} \left(\frac{3}{2} b^2 - \frac{R^2}{2} \right) 4\pi R^2 dR$$

$$= \frac{4\pi \rho^2 b^5}{15\epsilon_0}$$

$b \rightarrow 0$ energy goes to ∞
 (this is a mathematical point charge)

Strictly there are no point charges
 in ~~use~~ as much as the smallest
 charge unit, the electron is itself
 a distribution of charge.

Problem

(16)

Given the field $\vec{E} = \frac{k}{r} \hat{a}_r$ in cylindrical coords show that the work needed to move a point charge q from any radial distance to a point at twice the radial distance is independent of R

$$dW = -q \vec{E} \cdot d\vec{l} = -q E r dr = -\frac{kq}{r} dr$$

$$W = -kq \int_{r_1}^{2r_1} \frac{dr}{r} = -k \ln 2$$

independent of r_1

Problem For a line charge $\rho_l = \frac{10^{-9}}{2} \frac{C}{m}$

on the z axis find V_{AB} where $A = (2m, \frac{\pi}{2}, 0)$ and $B = (4m, \pi, 5m)$

$$V_{AB} = -\int_B^A \vec{E} \cdot d\vec{l} \quad \vec{E} = \frac{\rho_l}{2\pi\epsilon_0 r} \hat{a}_r$$

only radial $\vec{E} \cdot d\vec{l} \rightarrow E dr$

$$V_{AB} = -\int_4^2 \frac{10^{-9}}{2(2\pi\epsilon_0 r)} dr = -9 \ln r \Big|_4^2 = 6.24V$$

Problem Given a field

$$\vec{E} = -\left(\frac{16}{r^2}\right) \hat{a}_r \quad \frac{V}{m} \quad \text{is spherical coords}$$

find the potential of point $(2m, \pi, \pi/2)$
with respect to $(4m, 0, \pi)$

Equipotential lines are concentric spherical shells

$r = 2m$ surface A

$r = 4m$ surface B

$$V_{AB} = - \int_2^4 \frac{-16}{r^2} dr = -4V$$

Problem Find the potential at $r_A = 5m$

with respect to $r_B = 15m$ due to

a point charge $Q = 500 \mu C$

at the origin and zero reference @ infinity

$$V_{AB} = \frac{Q}{4\pi\epsilon} \left(\frac{1}{r_A} - \frac{1}{r_B} \right)$$

$$= \frac{500 \times 10^{-12}}{4\pi (10^{-9}/36\pi)} \left(\frac{1}{5} - \frac{1}{15} \right) = 0.6V$$

the zero reference is not needed



It is needed to find V_5 and V_{15}

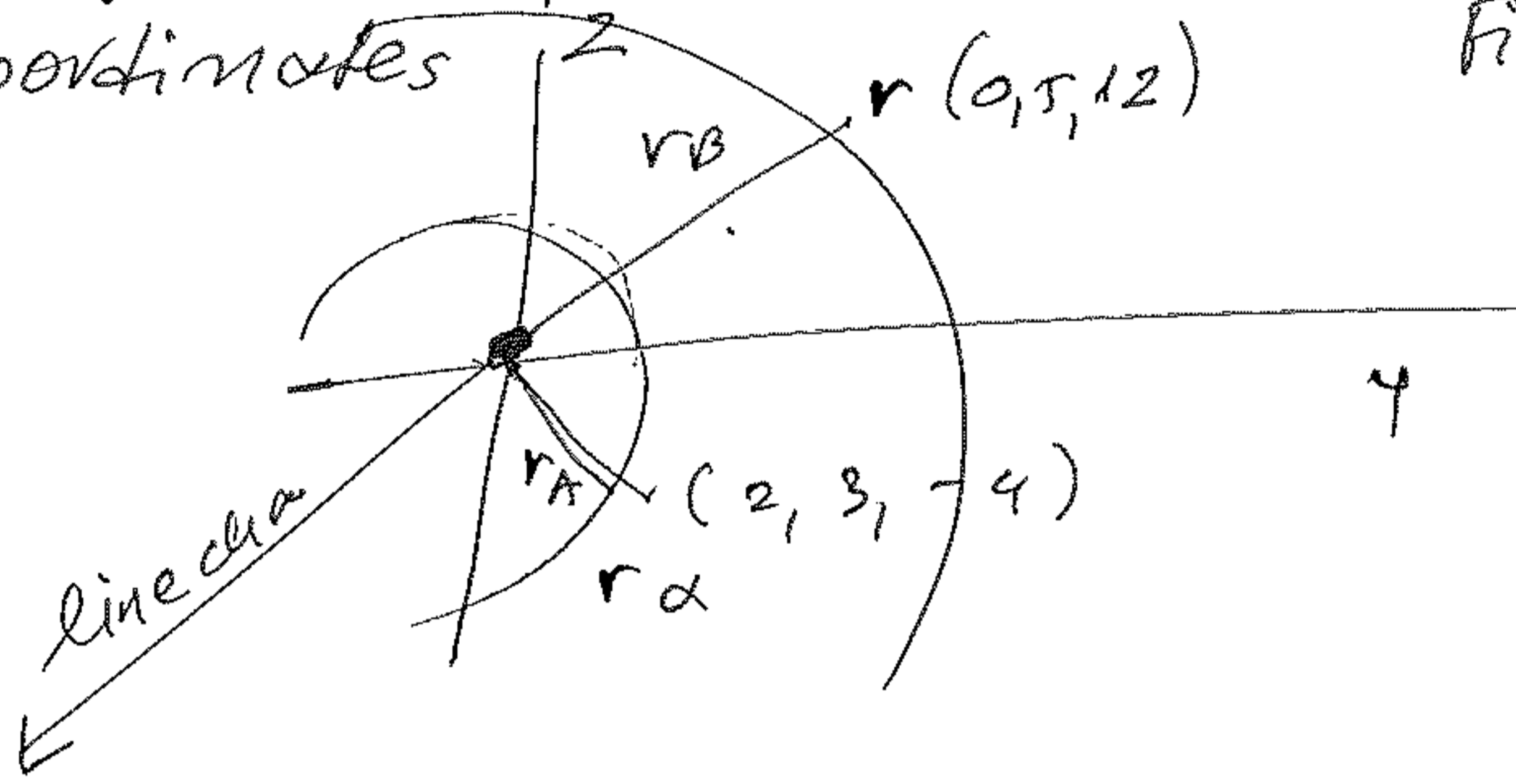
$$V_5 = \frac{2}{9060} \left(\frac{1}{5} \right) = 0.9 \text{ V}$$

$$V_{15} = \frac{2}{416} \left(\frac{1}{15} \right) = 0.3 \text{ V}$$

$$V_{AB} = V_5 - V_{15} = 0.6 \text{ V}$$

Problem

A line charge ρ_L lies along the x-axis and the surface of 0 potential passes through the point $(0, 5, 12)$ m in cartesian coordinates. Find the potential at $(2, 3, -4)$



$$r_A = \sqrt{9+16} = 5\text{m} \quad r_B = \sqrt{25+144} = 13\text{m}$$

$$V_{AB} = - \int_{r_B}^{r_A} \frac{\rho_L}{2\pi\epsilon_0 r} dr = - \frac{\rho_L}{2\pi\epsilon_0} \ln \frac{r_A}{r_B}$$

$$(\text{= } 6.88\text{V})$$



Problem

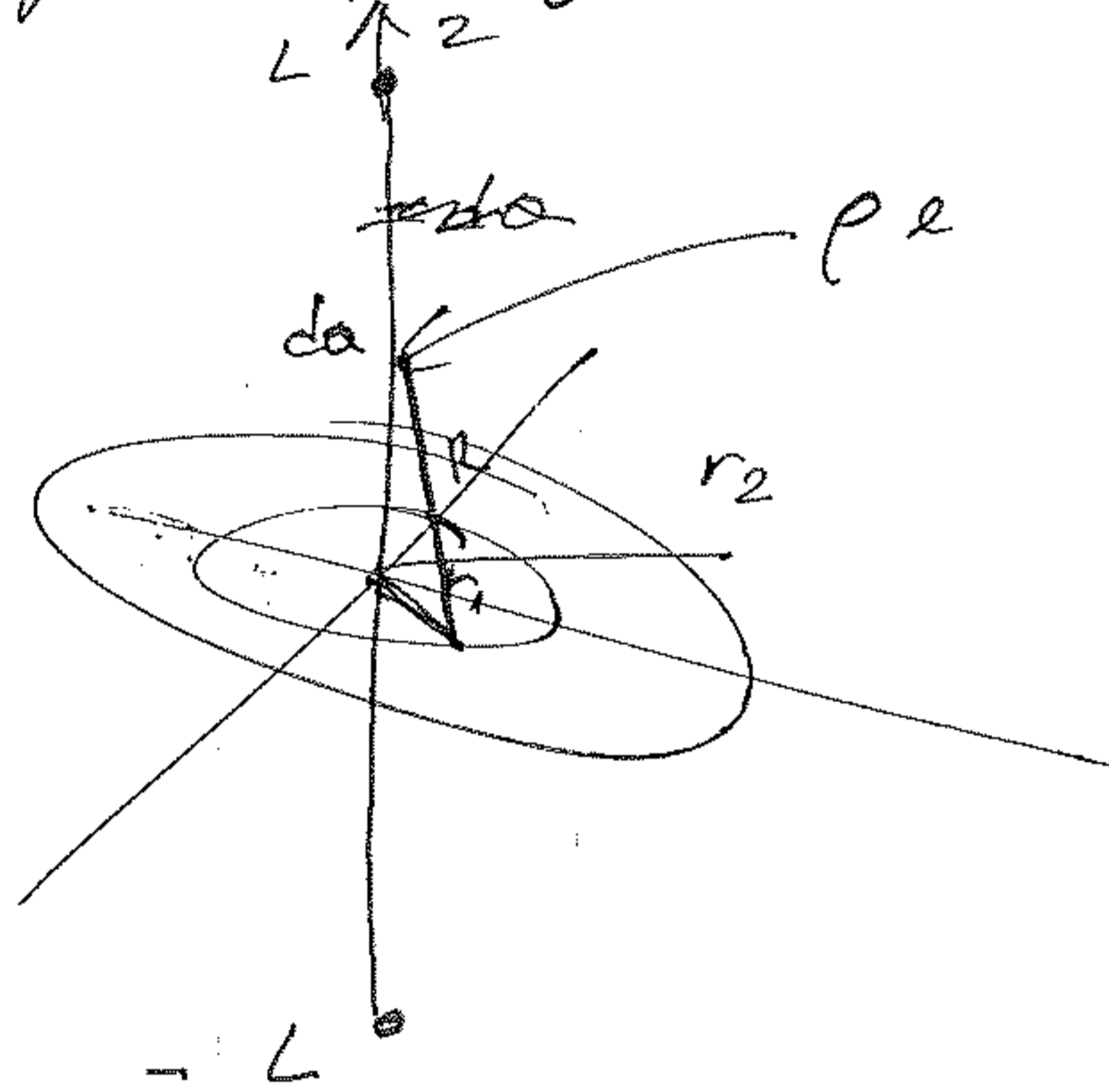
The electric field between two concentric cylindrical conductors at $r = 0.01 \text{ m}$ and $r = 0.05 \text{ m}$ is given by $\vec{E} = \left(\frac{10^5}{r}\right) \hat{a}_r \left(\frac{\text{V}}{\text{m}}\right)$ (fringing neglected)

Find the energy stored in a 0.5 m length assume free space

$$\begin{aligned}
 W_E &= \frac{1}{2} \int \epsilon_0 E^2 dV = \\
 &= \frac{\epsilon_0}{2} \int_0^{0.5} \int_0^{2\pi} \int_{0.01}^{0.05} \left(\frac{10^5}{r}\right) r dr d\phi dz \\
 & \quad (= 0.224 \text{ J.})
 \end{aligned}$$

Problem

Charge is distributed uniformly along a straight line of finite length $2L$



Show that for two external points near the midpoint such that r_1 and r_2 are small compared to the length L the V_{12} is the same as for infinite line charge

$$\text{Point } V_1 = \int_0^L \frac{\rho_e dz}{4\pi\epsilon_0 (z^2 + r_1^2)^{3/2}}$$

$$= \frac{\rho_e}{4\pi\epsilon_0} \left[\ln(z + \sqrt{z^2 + r_1^2}) \right]_0^L$$

$$= \frac{\rho_e}{4\pi\epsilon_0} \left(\ln(L + \sqrt{L^2 + r_1^2}) - \ln r_1 \right)$$

$$V_2 = \frac{\rho_e}{2\pi\epsilon_0} \left[\ln(L + \sqrt{L^2 + r_2^2}) - \ln r_2 \right]$$

$$L \gg r_1 \quad L \gg r_2$$

$$V_1 \approx \frac{\rho_e}{2\pi\epsilon_0} (\ln 2L - \ln r_1)$$

$$V_2 \approx \frac{\rho_e}{2\pi\epsilon_0} (\ln 2L - \ln r_2)$$

$$V_{12} = V_1 - V_2 \approx \frac{\rho_e}{2\pi\epsilon_0} \ln \frac{r_2}{r_1}$$



Problems

a spherical conducting shell of radius α centered at the origin has a potential field

$$V = \begin{cases} V_0 & r \leq \alpha \\ V_0 \frac{\alpha}{r} & r > \alpha \end{cases}$$

(with 0 reference @ infinity). Find an expression for the stored energy

$$\vec{E} = -\nabla V = \begin{cases} 0 & r < \alpha \\ V_0 \frac{\alpha}{r^2} \hat{r} & r > \alpha \end{cases}$$

$$W_E = \frac{1}{2} \int_{\text{vol}} \epsilon_0 E^2 du = 0 + \frac{\epsilon_0}{2} \int_0^{2\pi} \int_0^\pi \int_\alpha^\infty \left(\frac{V_0 \alpha}{r^2} \right)^2 r^2 \sin \theta dr d\theta d\phi$$

$$= 2\pi \epsilon_0 V_0^2 \alpha$$

The total charge on the shell is?

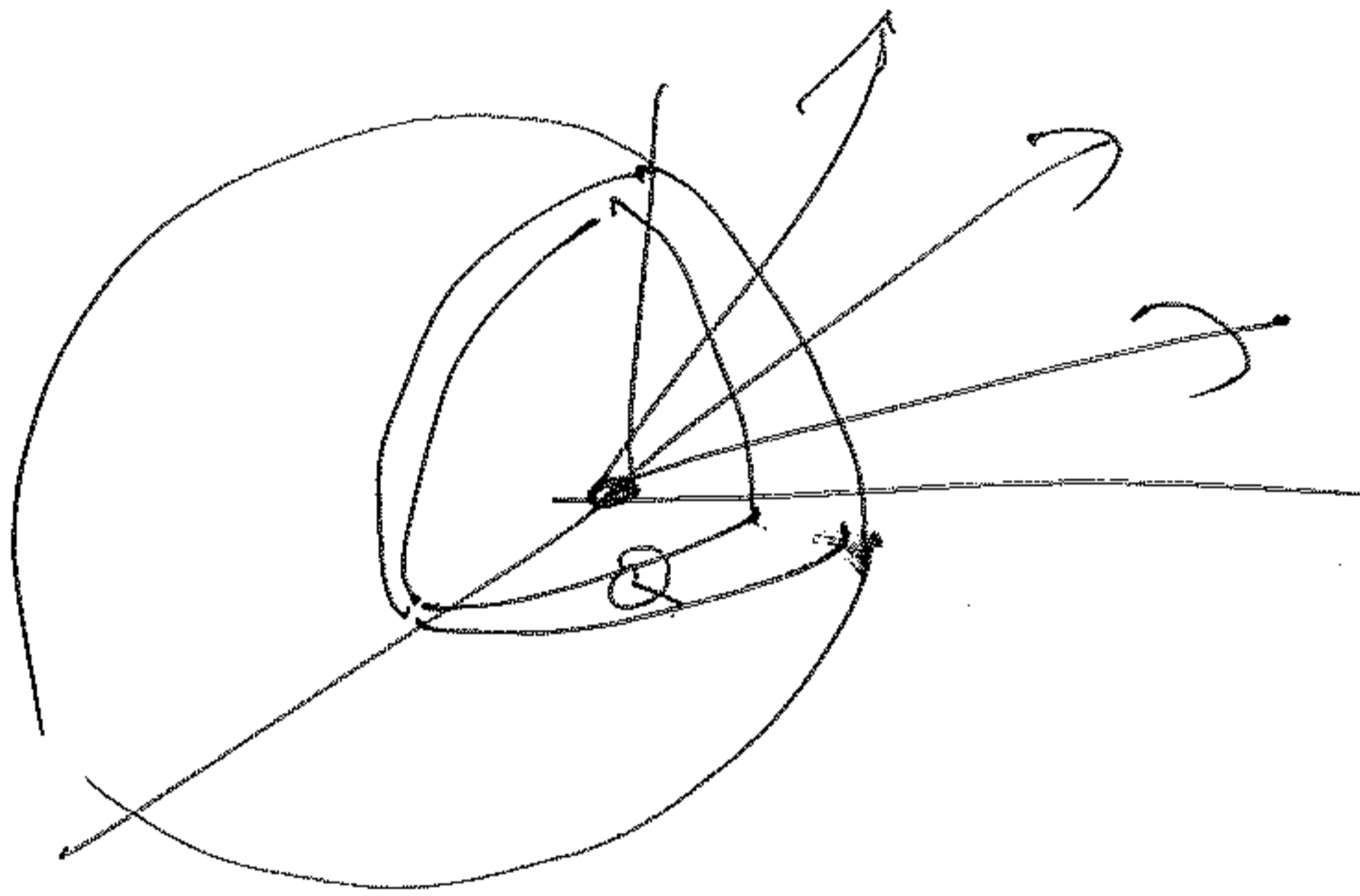
$$Q = \epsilon E A = \left(\frac{\epsilon_0 V_0 \alpha}{\alpha^2} \right) (4\pi \alpha^2) = 4\pi \epsilon_0 V_0 \alpha$$

the potential @ the shell is $V = V_0 \Rightarrow W_E = \frac{1}{2} QV$
energy stored in a spherical capacitor w/ other plate @ infinity

$\oint \vec{D} \cdot d\vec{S}$

Under static conditions the field outside a conductor is zero both tangential and normal components unless there exists a surface charge distribution.

A surface charge does not imply a net charge in the conductor.



Consider a positive charge Q at the origin of spherical coords. If the point charge is enclosed by an uncharged conducting ~~surface~~ spherical shell of finite thickness the field is

$$\vec{E} = \frac{+Q}{4\pi\epsilon_0 r^2} \hat{a}_r$$

except within the conductor where $\vec{E} = 0$