Gravitation-Induced Electric Field near a Metal*†

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A quantum-mechanical formalism is developed to calculate the electric field produced in the vicinity of a metallic object through the influence of the earth's gravitation. The field is proportional to the gradient of the ground-state energy eigenvalue of the object with respect to the position of a test charge located at the field point. This expression can be reduced to the solution of a problem in classical electrostatics, and is valid as well for a superconductor. Simple explicit results are obtained for the field within a closed metallic shell of arbitrary shape, and outside of a metallic sphere. In the former case, the field is uniform and equal to mg/e, directed so as to exert an upward force on an electron; m and e are the electron mass and charge, and g is the acceleration of gravity. This result is of importance in connection with current experiments on the free fall of electrons and positrons, and leads to the expectation that shielded electrons will not fall, while shielded positrons will fall with acceleration $2g$. Some comments are made on the gravitation-induced electric field near a nonconductor, and on the field near a rapidly rotating solid.

I. INTRODUCTION

CONSIDERATION of experiments now under way on the free fall of electrons and positrons[1] has raised the question whether or not the necessary metallic shield produces an electric field that affects the falling particles, because of the influence of the earth's gravitation on the metal.[2] It is apparent that each electron and nucleus in the metal must be acted on by an average electric field of such magnitude that it exactly balances its weight. Thus the quantum-mechanical expectation value of the electric field on an electron of mass m and charge $-e$ must be $-(mg/e)\hat{z}$, where g is the acceleration of gravity and $\hat{z}$ is a unit vector in the upward direction. Since the electrons occupy most of the volume, the metal is nearly filled with this field, which would then be expected to be present also within a shield having the form of a metallic shell. On the other hand, a nucleus of mass M and charge Ze experiences an average electric field $+(Mg/Ze)\hat{z}$, and it might well be asked if the presence of this field alters the earlier conclusions. It seems likely that it does not, since the nuclei are well localized and occupy a very small fraction of the total volume, and moreover are separated from the region outside the metal by conduction electrons.[3] Nevertheless, it seems worthwhile to see how the electric field outside of a metallic object can actually be calculated, when account is taken of gravitation and of the constraints that support the weight of the object.

We start in Sec. II with the full quantum-mechanical Hamiltonian for the interacting electrons and nuclei of the metal, and include the gravitational potential energy of the earth and the potential energy associated with the supporting constraints. The expectation value of the electric field at any point outside is easily expressed in terms of the ground-state eigenfunction of the metal. It is then shown that if a classical test charge is added to the system at the point at which the field is to be calculated, this field is proportional to the gradient of the ground-state eigenvalue of the new Hamiltonian with respect to the position of the charge. In this way the field is related in Sec. III to the changes in positions of the electrons and nuclei that are induced by the test charge, and hence to the solution of a problem in classical electrostatics. Simple explicit solutions are obtained in Sec. IV for the field within a closed metallic shell of arbitrary shape, and outside of a metallic sphere. A correction that arises from penetration of the electric field of the test charge into the metal is estimated in Sec. V, and shown to be negligible.

II. FORMULATION OF THE PROBLEM

The metal is assumed for simplicity to be monatomic, although the formalism is easily extended to alloys. The Hamiltonian of the object in the earth's gravitation may then be written H_0+H_g+V, where the three terms represent the isolated object, the gravitational interaction with the earth, and the supporting constraints, respectively. H_0 is the sum of the kinetic-energy operators for all the electrons (coordinates $\mathbf{r}_i$) and all the nuclei (coordinates $\mathbf{r}_\kappa$), together with the potential energy of interaction between them; it need not be written down explicitly. H_g is given by the expression

$$H_g=mg\sum_i z_i+Mg\sum_\kappa z_\kappa. \qquad (1)$$

A specific model must be chosen for V. We assume that the supporting constraints may be treated classically, and that they are elastic and nonconducting. The metallic object moves as a rigid body against these constraints, in such a way that motion in any direction develops a proportional restoring force in accordance

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[1] F. C. Wittenborn, Ph.D. thesis (unpublished) (available on microfilm or manuscript from University Microfilms, Inc., 313 N. First St., Ann Arbor, Michigan).
[2] F. C. Wittenborn, G. E. Hahne, and W. M. Fairbank (private communication).
[3] This qualitative view of the situation arose in the course of discussions with F. Bloch and W. Kohn, whom the authors wish to thank.

with a symmetric elasticity tensor. V is then a function of the $\mathbf{r}_i$ and $\mathbf{r}_\kappa$ which need not be specified. We shall only be concerned with the gravitation-induced electric field produced by the object itself, not the constraints. The field of the latter would have to be added in separately if the constraints are close enough to the field point to be of interest (see Sec. VI).

The operator for the electric field at the point $\mathbf{r}_0$ is

$$\mathbf{E}(\mathbf{r}_0)=-\sum_i \frac{e(\mathbf{r}_0-\mathbf{r}_i)}{r_{0i}^3}+\sum_\kappa \frac{Ze(\mathbf{r}_0-\mathbf{r}_\kappa)}{r_{0\kappa}^3}. \tag{2}$$

We wish to calculate the expectation value of this field operator for the stationary ground-state eigenfunction of the Hamiltonian H_0+H_g+V. Our procedure consists in adding a classical test charge q to the system at the point $\mathbf{r}_0$. This charge may be thought of as being fixed in position, and of having infinitesimal strength in the sense that the limit $q\rightarrow 0$ is ultimately to be taken. The total Hamiltonian is then $H=H_0+H_g+H_q+V$, where

$$H_q=-\sum_i \frac{qe}{r_{0i}}+\sum_\kappa \frac{qZe}{r_{0\kappa}}. \tag{3}$$

It follows from Eqs. (2) and (3) that

$$\mathbf{E}(\mathbf{r}_0)=-(1/q)[\boldsymbol{\nabla}_0,H], \tag{4}$$

where $\boldsymbol{\nabla}_0$ is the gradient operator associated with the coordinate $\mathbf{r}_0$.

Suppose now that we are able to find the stationary ground-state eigenfunction ψ of H that has the energy eigenvalue E. Both ψ and E depend parametrically on q and $\mathbf{r}_0$. The expectation value of Eq. (4) for this ψ is

$$\begin{aligned}\langle\mathbf{E}(\mathbf{r}_0)\rangle_\psi &= -\frac{1}{q}\int\bar{\psi}(\boldsymbol{\nabla}_0H-H\boldsymbol{\nabla}_0)\psi d\tau \\ &= -\frac{1}{q}\int[\bar{\psi}\boldsymbol{\nabla}_0(E\psi)-E\bar{\psi}(\boldsymbol{\nabla}_0\psi)]d\tau \\ &= -\frac{1}{q}\int\bar{\psi}(\boldsymbol{\nabla}_0E)\psi d\tau \\ &= -(1/q)\boldsymbol{\nabla}_0E,\end{aligned} \tag{5}$$

where the integration is over all the $\mathbf{r}_i$ and $\mathbf{r}_\kappa$, but not over $\mathbf{r}_0$. The last step follows since $\boldsymbol{\nabla}_0E$ is a c number that can be taken outside of the integral, and ψ is normalized to unity.

In the limit $q\rightarrow 0$, ψ approaches the ground-state eigenfunction of H_0+H_g+V, so that the left side of Eq. (5) approaches the desired expectation value of the electric field at $\mathbf{r}_0$. Also, from general perturbation-theoretic arguments, the $\mathbf{r}_0$-dependent part of E is proportional to q for small q. Thus the right side of Eq. (5) approaches a well-defined limit as $q\rightarrow 0$. We are interested in the electric field produced by the earth's gravitation, and this field is expected to be proportional to g for small g. Thus we wish to calculate the part of the energy eigenvalue E that is proportional to gq, and then substitute this into Eq. (5) to find the gravitation-induced electric field.

III. REDUCTION TO A PROBLEM IN CLASSICAL ELECTROSTATICS

The gq-proportional part of E may be found by treating H_g and H_q as perturbations, both to first order. It is convenient to omit H_g initially and work with the Hamiltonian H_0+H_q+V, and then to include H_g to first order. We refer to the ground-state eigenfunctions of H_0+V and of H_0+H_q+V as ψ_0 and ψ_q, respectively; both are appropriately symmetrized and normalized to unity. Electron and nuclear densities (numbers per unit volume) may be defined for each eigenfunction:

$$\begin{aligned}\rho_{0,q}^{(e)}(\mathbf{r}) &= \int|\psi_{0,q}|^2\sum_i \delta^3(\mathbf{r}-\mathbf{r}_i)d\tau, \\ \rho_{0,q}^{(N)}(\mathbf{r}) &= \int|\psi_{0,q}|^2\sum_\kappa \delta^3(\mathbf{r}-\mathbf{r}_\kappa)d\tau.\end{aligned} \tag{6}$$

The relation between the densities calculated with and without H_q may be written

$$\rho_q^{(e,N)}(\mathbf{r})=\rho_0^{(e,N)}(\mathbf{r})+q\rho_1^{(e,N)}(\mathbf{r})+0(q^2).$$

Uncharged Metallic Object

If we assume that the object is electrically neutral, then the only charge it contains is the surface-charge density induced by q, which may be calculated from classical electrostatics. In this case, the force exerted on the object by q is proportional to q^2, and the difference between ψ_q and ψ_0 that arises from motion of the object against the elastic constraints represented by V is also proportional to q^2. It then follows from Eqs. (6) that this contribution to $\rho_q^{(e,N)}(\mathbf{r})$ is of order q^2, so that V does not contribute to $\rho_1^{(e,N)}(\mathbf{r})$.

Insofar as the penetration of the electric field of q into the metal can be neglected,[4] the nuclei are shielded from this field by the conduction electrons. Thus the nuclear density change $\rho_1^{(N)}(\mathbf{r})$ caused by q is zero. The electron-density change $\rho_1^{(e)}(\mathbf{r})$ is proportional to the induced surface-charge density referred to above, which is classically calculable.

We now wish to calculate the first-order change in the ground-state energy eigenvalue when H_g is included; this is simply the expectation value of H_g for the eigenfunction ψ_q. There is of course a difference between ψ (the eigenfunction of the total Hamiltonian) and ψ_q that is of first order in g, since the object moves against the elastic constraints when gravitation is included.

[4] This point is considered in Sec. V, where the neglect of the penetrating field is justified.

However, the corresponding change in the energy eigenvalue is of order g^2, and hence can be neglected.

Since H_g, given in Eq. (1), is a sum of single-particle terms, its expectation value may be simply expressed in terms of the densities (6):

$$\int \bar{\psi}_q H_g \psi_q d\tau = mg\int z\rho_q{}^{(e)}(\mathbf{r})d^3r + Mg\int z\rho_q{}^{(N)}(\mathbf{r})d^3r. \quad (7)$$

The gq-proportional part of this is obtained by substituting $q\rho_1{}^{(e,N)}(\mathbf{r})$ for $\rho_q{}^{(e,N)}(\mathbf{r})$ in Eq. (7), to obtain

$$mgq\int z\rho_1{}^{(e)}(\mathbf{r})d^3r. \quad (8)$$

Charged Metallic Object

Suppose now that the object has a total charge Q, and the supporting constraints are again nonconducting and elastic. When g and q are zero, this charge produces an electric field at $\mathbf{r}_0$, which we call $\mathbf{E}_Q(\mathbf{r}_0)$. Inclusion of H_q gives rise to the force

$$-q\mathbf{E}_Q(\mathbf{r}_0)+0(q^2)$$

exerted on the object by q. Thus the difference between ψ_q and ψ_0 that arises from the motion of the object against the constraint potential V is now of order q, instead of q^2 as before. This means that in addition to the electron-density change $\rho_1{}^{(e)}(\mathbf{r})$ that is proportional to the induced surface-charge density mentioned in the preceding subsection, there is also a contribution to $\rho_1{}^{(e,N)}(\mathbf{r})$ from displacement of the object as a whole.

We call this displacement $\mathbf{d}_Q$; with elastic constraints, which may be represented by a symmetric tensor $\mathfrak{J}$, we have

$$\mathbf{d}_Q = -q\mathfrak{J}\cdot\mathbf{E}_Q(\mathbf{r}_0).$$

Then the part of the expectation value (7) of H_g that arises from this contribution to $\rho_1{}^{(e,N)}(\mathbf{r})$ is

$$M_0 g\hat{z}\cdot\mathbf{d}_Q = -M_0 gq\hat{z}\cdot\mathfrak{J}\cdot\mathbf{E}_Q(\mathbf{r}_0), \quad (9)$$

where M_0 is the total mass of the object. Substitution into Eq. (5) should give the change in the electric field at $\mathbf{r}_0$ that is caused by the combination of the earth's gravitation, the constraints, and the charge Q. That this is indeed the case may be seen by noting that when gravitation is introduced, the object is displaced by the vector

$$\mathbf{d}_g = -M_0 g\mathfrak{J}\cdot\hat{z}.$$

The consequent change in electric field at $\mathbf{r}_0$ is

$$-(\mathbf{d}_g\cdot\nabla_0)\mathbf{E}_Q(\mathbf{r}_0) = M_0 g(\hat{z}\cdot\mathfrak{J}\cdot\nabla_0)\mathbf{E}_Q(\mathbf{r}_0), \quad (10)$$

since $\mathfrak{J}$ is symmetric. When account is taken of the fact that $\nabla_0\times\mathbf{E}_Q(\mathbf{r}_0)=0$, it is easily seen that this is equal to the result obtained by substituting (9) into (5).

The preceding discussion shows that our calculational procedure yields the correct but uninteresting result (10) for that part of the gravitation-induced electric field that arises from Q, and hence provides a useful check on the formalism. The interesting part of the field is given by substitution of (8) into (5); we now consider two examples.

IV. EXPLICIT SOLUTIONS

The gravitation-induced electric field can be calculated in terms of the surface charge density $\sigma(\mathbf{r})$ induced on the object by the test charge q. Once $\sigma(\mathbf{r})$ is known, the electron-density change may be found from the relation

$$-eq\rho_1{}^{(e)}(\mathbf{r})d^3r = \sigma(\mathbf{r})dA,$$

where dA is a differential element of surface area. Substitution into Eqs. (8) and (5) then shows that the expectation value of the electric field may be obtained from an appropriate electrostatic potential, in accordance with the relations

$$\langle\mathbf{E}(\mathbf{r}_0)\rangle_\psi = -\nabla_0\phi(\mathbf{r}_0),$$
$$\phi(\mathbf{r}_0) = -\frac{mg}{eq}\int z\sigma(\mathbf{r})dA. \quad (11)$$

Field within a Closed Metallic Shell

Let $\mathbf{E}_q(\mathbf{r})$ be the electric field at the point $\mathbf{r}$, that is produced by the test charge q at a point $\mathbf{r}_0$ that lies within a closed metallic shell of arbitrary shape. When $\mathbf{r}$ is on the inner surface of the shell, $\mathbf{E}_q(\mathbf{r})$ is perpendicular to the surface, and the surface-charge density is given by the relation

$$4\pi\sigma(\mathbf{r})dA = -\mathbf{E}_q(\mathbf{r})\cdot\mathbf{dA}, \quad (12)$$

where the vector $\mathbf{dA}$ is along the outward normal. The total charge associated with this $\sigma(\mathbf{r})$ is $-q$. There is also a surface-charge density induced on the outer surface of the shell, which integrates to q. However, its distribution over the outer surface is independent of the position $\mathbf{r}_0$ of the test charge. Thus it only contributes an additive constant to $\phi(\mathbf{r}_0)$, and can be ignored.

Gauss' theorem may be applied to any vector $\mathbf{W}$ in the form

$$\int(\nabla\cdot\mathbf{W})d^3r = \int\mathbf{W}\cdot\mathbf{dA},$$

where the two integrals are over the entire inner volume and inner surface of the shell, respectively. With the substitution $\mathbf{W}=z\mathbf{E}_q$, Gauss' theorem gives

$$\int(E_{qz}+z\nabla\cdot\mathbf{E}_q)d^3r = -4\pi\int z\sigma(\mathbf{r})dA, \quad (13)$$

where use has been made of Eq. (12). Now $\nabla\cdot\mathbf{E}_q$ is equal to zero except at $\mathbf{r}_0$, and its volume integral over a region that includes this point is $4\pi q$. Thus the second term on the left side of Eq. (13) is $4\pi q z_0$. The first term on the left side may be written $\int\int dxdy \int E_{qz} dz$. The z integral is zero, at least for paths of integration that do not pass through the point $\mathbf{r}_0$, since the inner surface is an equipotential; it can be shown without difficulty that the point $\mathbf{r}_0$ is not in fact exceptional.

Thus Eqs. (11) and (13) show that $\phi(\mathbf{r}_0)=(mg/e)z_0$, and hence that

$$\langle \mathbf{E}(\mathbf{r}_0)\rangle_\Psi = -(mg/e)\hat{z} \tag{14}$$

everywhere within the shell. This result is in agreement with the qualitative discussion given in Sec. I.

Field outside of a Metallic Sphere

The charge density induced at a point on the surface of a metallic sphere of radius R by a charge q at a distance r_0 from the center $(r_0>R)$ is

$$\sigma(\theta)=\frac{q}{4\pi R}\left[\frac{1}{r_0}-\frac{r_0^2-R^2}{(r_0^2+R^2-2r_0R\cos\theta)^{3/2}}\right],$$

where θ is the angle between the radial lines to q and to the point on the surface. The total charge associated with this $\sigma(\theta)$ is zero. The integration in the second of Eqs. (11) is readily performed, and leads to the potential

$$\phi(\mathbf{r}_0)=mgR^3\cos\theta_0/er_0^2,$$

where θ_0 is the angle between the radial line to the point $\mathbf{r}_0$ and the upward direction. This is just the potential that would be produced by an electric dipole of moment mgR^3/e located at the center of the sphere and oriented along the positive z axis. It is also the potential that would be obtained by matching an exterior solution of Laplace's equation to the potential that corresponds to the uniform field $-(mg/e)\hat{z}$ inside the sphere.[4a]

V. PENETRATION OF THE ELECTRIC FIELD

It was assumed in Sec. III that the nuclei are shielded from the field of the test charge q by the conduction electrons. There will be, however, some penetration of the electric field, which will both displace and polarize the ions (nuclei plus tightly bound electronic clouds). In order to gauge the importance of this effect, we must estimate the ratio of the contributions for a typical element dA of surface area of the two terms on the right side of Eq. (7).

Both of these contributions are proportional to E_q, the electric field on the surface produced by q. The electron term may be written

$$mgl\sigma dA/e = mglE_q dA/4\pi e, \tag{15}$$

where l is a typical vertical height through which the electrons that make up the surface-charge density σ have been displaced.

The ion-displacement part of the nuclear term may be estimated by first assuming an attenuation factor β such that the sum of the electric fields penetrating to all layers of ions is βE_q. Next, we assume that each ion has an effective charge $Z'e$ and a restoring force constant K, so that the sum of the deflections of all layers of ions is $\beta E_q Z'e/K$. The number of ions in each layer in the element dA of surface area is $N^{2/3}dA$, where N is the number density of ions. Thus if this area element is horizontal, the contribution to the second term of (7) from ion displacement has its greatest value and is equal to

$$Mg\beta E_q Z'eN^{2/3}dA/K. \tag{16}$$

K may be estimated from the Einstein temperature θ_E of the metal by means of the expression $K=M(k\theta_E/\hbar)^2$, where k is Boltzmann's constant and $\hbar$ is Planck's constant divided by 2π.

The ion-polarization part of the nuclear term may be estimated by assuming that the induced dipole moment results entirely from displacement of the nucleus with respect to the center of charge of the electronic cloud. Then the sum of the deflections of all layers of nuclei is $\beta E_q\alpha/Ze$, where α is the ionic polarizability. The maximum contribution to the second term of (7) from ion polarization is then

$$Mg\beta E_q\alpha N^{2/3}dA/Ze. \tag{17}$$

The ratio of (17) to (16) is $\alpha K/ZZ'e^2$. For copper,[1] with $\theta_E\approx 240°$K, we obtain $K\approx 10^5$ in cgs units. A reasonable value for α is 2×10^{-24} cm^3, and Z' is expected to be between 1 and 2. We then find that this ratio is roughly equal to 0.02. Thus, ion polarization can be neglected in comparison with ion displacement.

The ratio of (16) to (15) is $4\pi Me^2\beta Z'N^{2/3}/mKl$. For copper, with $N=8.45\times10^{22}$ atoms/cm^3, we find that this ratio is roughly equal to $(\beta/l)\times10^{-2}$. Since β is expected to be appreciably less than unity, and l is at least of order 10 cm in a typical experiment, field penetration can safely be neglected.

VI. CONCLUDING REMARKS

The principal conclusion of this paper follows from Eq. (14): that free electrons are not expected to fall under gravity if they are within a closed metallic shell of arbitrary shape, since their weight is exactly balanced by the gravitation-induced electric field produced by the metal. In similar fashion, free positrons, if they have normal gravitational properties, should fall with acceleration $2g$. In actuality, the metallic shell does not have to be completely closed in order for these results to be valid to good approximation. Explicit calculations for other configurations, such as a long

[4a] *Note added in proof*. It can be shown from Eqs. (11) that the latter procedure is valid in general, not just for a sphere. This provides an alternate treatment for metallic objects of arbitrary shape, and incidentally simplifies the derivation of Eq. (14).

vertical cylindrical tube with open ends, could be carried out if warranted by an experimental situation. It should also be noted that there is nothing in the present calculation that precludes its applicability to superconducting objects.[4b]

One aspect of the role of the supporting constraints was alluded to early in Sec. II, but not discussed further. This is the fact that there is a gravitation-induced electric field associated with them as well as with the object that they support. If the object is a closed metallic shell and the constraints are on the outside, it is apparent that they have no effect on the field inside. In general, if the constraints are themselves metallic, they may be considered as extensions of the object, and sufficiently remote parts contribute a negligible amount to the field. However, the field around the sphere of Sec. IV will depend to some extent on the size, shape, and orientation of the constraints.

While the general theory developed in this paper is applicable to nonconducting objects and constraints as well, the calculation would differ considerably in detail. Since it seems likely that stray charges would greatly complicate any measurement of the gravitation-induced electric field near a dielectric, no attempt has been made to obtain a theoretical result. There is, however, one situation that can be approached in a straightforward manner: that in which various metallic parts of the object and constraints are separated by thin nonconducting wafers. Their direct contribution to the field can be made very small by making the wafers very thin, and they then serve merely to permit the metallic parts to be at different potentials under the influence of the test charge, and to prevent the flow of induced charge from one part to another.

It is natural to think of enhancing the gravitation-induced electric field by substituting for gravitational force the much larger centrifugal force that can be obtained by rapid rotation. However, we have been unable to obtain a treatment as simple and general as that presented here, in the case of a rotating solid. This does not perhaps seem surprising when it is remembered that a rotating superconductor has qualitatively different properties from a rotating normal metal,[5] while as we have seen they have the same behavior with respect to gravitation when at rest.

[4b] *Note added in proof.* For a discussion of a related phenomenon, see B. S. DeWitt, Phys. Rev. Letters **16**, 1092 (1966).

[5] F. London, *Superfluids* (Dover Publications, Inc., New York, 1961), Vol. 1, p. 78.

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Primary Cosmic-Ray Spectrum at High Energies and Spectra of γ Rays and Muons in the Atmosphere*

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The shape of the primary cosmic-ray spectrum at very high energies and its connection with the spectra of high-energy γ-rays and muons in the atmosphere have been discussed. Using a simple formulation for calculating secondary spectra arising from power-law primary spectra cutoff at arbitrary points, it is shown that a specific two-component model for the primary spectrum explains the present observations on the high-energy γ-ray spectrum at all altitudes and the muon spectrum at sea level, without invoking any change in the character of the interactions at high energies. Such a spectrum is also consistent with air-shower observations.

1. INTRODUCTION

ABOVE 100 GeV the methods which have been so far used for measuring the energy of primary cosmic-ray particles are essentially calorimetric in character. For energies greater than $\sim 10^{14}$ eV air showers are used, while below this the current information comes from the study of the energy going into the soft component in interactions produced by a primary particle in heavy-metal emulsion assemblies and arrangements using absorbers and ionization chambers. Transforming a measured particle energy spectrum into an energy-per-nucleon spectrum requires additional information on the chemical composition of the primary nuclei as a function of energy, which is usually not available. However, the following statements may be made regarding the chemical composition:

(i) The relative number of high-energy interactions produced by primary particles of different charges in large emulsion assemblies exposed at the top of the atmosphere suggests that, at least up to 10^{13} eV/nucleon, the primary composition is not very different from that found at low energies.

* A preliminary version of this paper was presented at the International Cosmic Ray Conference in London in 1965.