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Ph1b Section 05

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Outline

Principles

Rotations

Logistics

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Principles

The principle of special relativity & the principle of the constancy or invariance of the speed of light

- All reference frames in rectilinear, uniform and irrotational motion, i.e. the so-called **inertial reference frames** shall be completely equivalent in physics. No inertial frame shall be distinguished from any other inertial frame by any property.
- The speed of light in the vacuum has the same value in each inertial frame, irrespective of the velocities of the light source or the light receiver. **it is a fundamental physics constant c=299,792,458 m/s**
- **March challenge (4U): write an essay titled "if $c = 45$ km/h the world would look like this:" (you will need to be quantitative)**

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Lorentz transformations as rotations

- we have considered cases with IRF with axes parallel to each other in relative motion to the x, x' direction.
- the corresponding Lorentz tranforamtion involves two variables x and ct into x' and ct'
- the transformation is linear and conserves the interval

$$s'^2 = c^2t'^2 - x'^2 = c^2t^2 - x^2 = s^2$$

- if instead of minus we had a plus sign we would say that $s = s'$ is the distance of the corresponding point fromt he origin.

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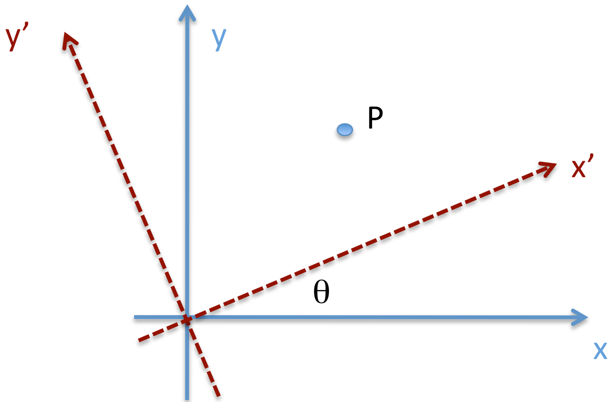
Lorentz transformations as rotations

- A transformation in a 2D space that involves both coordinates and is lineae and conserves distances is **rotation**
- so in fact as alluded so far

$$s'^2 = c^2t'^2 - x'^2 = c^2t^2 - x^2 = s^2$$

is a spacial kind of 2D "distance" and the Lorentz tranformations are rotations in this space.

Lorentz transformations as rotations



Consider the Cartesian coordinate system x, y and the rotated by θ x', y'. Find the coordinates of P in the two systems.

Lorentz transformations as rotations

$$x' = \cos\theta x + \sin\theta y$$

$$y' = -\sin\theta x + \cos\theta y$$

or in matrix form

$$\begin{pmatrix} x' \\ y' \end{pmatrix} = \begin{pmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix}$$

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express the sin and cos in terms of tangent of the same angle (trigonometry please)

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now lets set $y = ct$ so that we get the time coordinate and change the order of the variables: instead of (x, ct) we will do (ct, x).

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$$\beta = \frac{v}{c}, \quad \gamma = \frac{1}{\sqrt{1-\beta^2}}$$

some similarity but major caveat the minus signs.

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rotation with a funny sign

- the difference is that the transforamtion matrix is symetrical for the Loretz transformation but antisymmetrical for the Euclidean roatation
- if instead of ratating both axes in the same direction (tru rotation), we rotate them in opposite direction : i.e rotate x counterclockwise and ct clockwise: The resulting coordinate system (ct', x' will be skewed rather than rotated.
- from matehtical viewpoint the coordinate transformation can be seen as a rotation with a funny sign (the sign difference in non-diagonal elements of the transformation matric will dissapear).

one more minus sign difference

The difference of the other minus sign is because in the Euclidean rotation

$$r^2 = x^2 + y^2 = x'^2 + y'^2$$

as opposed to the Lorentz transformation that preserves

$$s'^2 = c^2 t'^2 - x'^2 = c^2 t^2 - x^2 = s^2$$

Outline

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Office 265 Lauritsen, x2471, x6676, x6667
Notes and other material/workbooks references etc will be posted in the 05 Section twiki
https://twiki.hep.caltech.edu/twiki/bin/view/Main/Smaria
(starting next week)
A student twiki will be set there for you (and an account)
Dr. Dorian Kcira dkcira@caltech.edu is managing the twiki and will be sending you info on the account

Reference Frames and Coordinate systems

- Reference frame is usually (but not always) a physical rather rigid object to which we refer our measurements and observations (car train plane, spaceship, the earth, the galaxy, even a cluster of galaxies etc)
- A Coordinate system is a way we specify a position by assigning to it a set of numbers (Cartesian, spherical, cylindrical etc); Geometrically thay can be represented as a triad of unit vectors $\hat{\vec{x}}$, $\hat{\vec{y}}$, $\hat{\vec{z}}$. A point in space is specified by the orthogonal projections of its position vector onto the corresponding directions.
- There are infinite such triads we can devise. They are all distinct and they can all be obtained from another by appropriate rotations and/or reflections.
- A reference frame and a coordinate system are different concept and specifically the former does not specify the latter

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Newtonian space and time (from Principia)

- Absolute space in its own nature, without regard to anything external, remains always similar and immovable
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